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Digital stability analysis of power generating units

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DIGITAL STABILITY ANALYSIS OF POWER GENERATING UNITS

by

Joseph Donald Musil

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

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Signature was redacted for privacy.

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Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Ames, Iowa

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I. INTRODUCTION

The purpose of this investigation is to study the dynamic stability of a typically large (700 to 800 mva) synchronous generator unit with its automatic voltage regulator and governor when connected to an infinite bus through a typically long transmission line. Sets of higher-order differential equations describing the dynamical behavior of the various elements are formulated by state-space methods into sets of first-order equations. In matrix algebra the equations take the form $\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{D}\,\underline{u}$ where $\underline{A}$ and $\underline{D}$ are the matrices of constant coefficients, $\underline{u}$ is the forcing function input vector and $\underline{x}$ is the vector representing the state of the system variables. Stability is determined by digital methods in this investigation by insuring that the real part of the eigenvalues of the A-matrix at various states remains negative.

The voltage at the infinite bus is held at 1.0 per-unit (pu). The generator terminal voltage is iterated to the value needed to supply various power outputs and power factors of operation at the generator terminals. The variations of tie-line reactances and system damping are investigated.

The excitation control system is of a currently standardized type. A derivative power damping signal is used. The effect of varying the voltage regulator parameters and the addition of the governor system on stability are investigated.

Addition of a tie-line control signal to the governor and the extension of the method of investigation to a multi-machine system are discussed briefly.

II. REVIEW OF LITERATURE

For many years the network analyzer was the tool used to compute the steady-state as well as transient stability by establishing new levels of operation in system load flow. These basic concepts, given in Kimbark (19), Crary (9), and by Westinghouse (33) produce conservative results but give a sound background on which to add the refinements present-day computers permit.

Parks (24,25) established the direct and quadrature component presentation of the synchronous machine. Kilgore (18) and Doherty and Nickle (10) by calculation and testing, clearly defined and established the accuracy of the reactances of the machine. Rankin (26,27) categorized approaches on per-unit notation. These have helped form the basis for a systematized approach enjoyed in stability studies today.

Stability studies limited by the computation facilities available, were soon enlarged upon. Concordia's work (7) in 1944, studying the steady-state stability of the synchronous machine with the voltage regulator, is a classic reference. Heffron and Phillips (14) in 1952 undertook the study of operation in under-excited regions, made possible by the use of faster, rotating voltage regulators. Improvements in excitation control systems has led to lower short circuit ratio generators with more mega-volt-ampere capacity from a given frame size. The excitation control systems which have been developed have been standardized (16) for system stability studies.

Trends mentioned above could only proceed as fast as computer facilities and programs evidenced stability. The November, 1964 Northeast power failure (13) caused new interest and concern in assaying the correctness of programs with respect to interconnected systems. Fortunately computer facilities were available.

Olive (23) in his program in 1965 gave one of the first papers on the problem of detailed programming of transient and steady-state stability permitting 500 machines, 1000 busses, and 1500 lines. Young and Webler (34) and their similar program permitted 250 machines, 1000 busses, and 2000 lines. Both programs contained the machine, control, and system representations deemed needful for these studies. Steady-state stability checks in these programs were accomplished by a step-input perturbation followed by a real-time run of five seconds or more to insure return to stable operation. Certain simplifications in transient stability programming were permissible for steady-state checks but the method of multi-step, swing-curve time solutions along with intermediate iterative load-flow adjustments are fundamental to this method. Many non-linearities can be accommodated. This method is costly but still presently necessary.

A large scale study of the current type in the paper by Lokay and Bolger (22) incorporates transient saliency, saturation, machine and system damping, speed governor system and excitation system with a 660 mw turbine-generator set. Another informative study by Ewart, Landgren, Temoshok, and Walkey (11) investigates a 532 mw cross-compound unit. Damper winding effect is included. Concordia (6) notes in particular the inclusion of damper winding effect is most critically needed in the case of cross-compound generators. Byerly, Skooglund, and Keay (4) in an analog study reported improvement in stability obtained with a power damping control signal fed into the voltage regulator.

VanNess (31) used modern control theory, root loci approach, in his investigation of interconnected systems incorporating the turbine, generator, governor and tie-line controller. Laughton (21) took Heffron's modeling of the synchronous generator and its A-constants combining it into a state-space representation of a 30 mw generator and regulator. The dynamic stability was checked by noting the negativeness of the real part of the eigenvalues of the A-matrix from the state-space formulation of the differential equations.

Undrill (30) extends the work of Laughton analytically and in particular separating out the turbine damping and rotor damping effects. A four-machine system is analyzed with methods given for extending to a higher number of machines.

Kekela and Firestone (17) present a study of under-excited operation, a problem associated with medium to long length transmission lines.

Schleif and White (28) give an analog study on oscillations prevalent between basically hydro and steam generating areas interconnected by long transmission lines.

III. SYSTEM STABILITY BY EIGENVALUES

Modern control theory (3) has permitted the investigation of stability of differential equations describing a system by means of root locus, Bode, or Nyquist plots, to name a few. Routh criterion approach determines stability by ascertaining the negativeness of the real part of the roots of the higher-order characteristic equation without factoring it.

A. Method of Analysis

The method employed to determine stability is to form the equations interrelating circuit values and motion as a series of first order differential equations. By means of Taylor's expansion, linearization around steady-state operating conditions is made and differential equations formed with the equations interrelated by variables defined as states of the vector space method (2). Placing these equations in matrix algebra notation (5) the required eigenvalues of such a reduced system matrix are the same as the roots of the characteristic equation. The negativeness of the real part of the eigenvalue assures stability; the more negative, the sooner the contribution of the associated term will die out. A complex conjugate root indicates oscillation with the higher frequencies associated with the larger imaginary parts.

B. Formulation of the Equation

In matrix algebra, the equations take the form $\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{D}\,\underline{u}$ where $\underline{A}$ and $\underline{D}$ are the matrices of constant coefficients, $\underline{u}$ is the forcing function input vector and $\underline{x}$ is the vector representing the state of the system variables. In this investigation the stability of the equations was determined by digital methods on an IBM 360/65 using Fortran admitting complex algebra programming. The real part of the eigenvalues of the A-matrix at various states must remain negative to insure that the solution equation decays to a stable final value as would be obtained in a real-time solution for x(t). Thus the linearized equations thus formed give a good approximation to the resulting time solutions. It is this piecewise or indirect method of determining stability that is investigated further.

General information concerning the boundaries of stable regions is not gained from this method but the stability limits under a multitude of operating conditions can be accurately determined.

C. Eigenvalue Routine

The correctness of the investigation relies heavily on the eigenvalue solution routine for a non-symmetric matrix by the QR transform by Imad and VanNess (15). This is available as a Share program library routine. The main program calls this library routine and the eigenvalues only were printed out in real and imaginary parts. Lacking a negative sign, the positive real part of any eigenvalue marks that particular power output and power factor setting, unstable.

The Share program was checked against the Krylov Method for non-symmetric matrices in Faddeeva* and checked to five decimal places. It is known, however, that the QR transform routine is dependent upon the A-matrix itself. The routine was dimensioned at 30 by 30.

D. Extent of the Investigation

A large modern single-shaft steam turbine driven (600 to 700 mw) generator was investigated. It was connected to an infinite bus through a typical long transmission line or tie line. It had an excitation control system and a prime-mover and governor system. The investigation follows Laughton (21) in per-unit on the machine base but varies in five ways:

1. The infinite bus is held at 1.0 pu on the machine base voltage and the effect of regulating by adjusting the generating terminal voltage is noted. Laughton held the terminal voltage at 1.0 pu.
2. The generator is large, typical of present day large units, whereas Laughton's generator was 30 mw.

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*Faddeva (12, p. 156).

3. Laughton investigated stability with the voltage regulator and machine only. This investigation includes the prime mover and governor.
4. Effect of varying machine damping, D, is included.
5. Effect of varying the tie-line reactance, X-line, is included. A Pi-line equivalent network (see Figure 2) is used.

The method of extending the investigation to include a tie-line controller is given; also, Laughton's method of extending the study to a multi-machine system in included.

IV. SYNCHRONOUS GENERATOR

The phasor diagram covering the Parks direct and quadrative-axis representation of the synchronous generator (25, 26) is given in Figure 1. It is convenient to include the tie-line connecting the generator to the infinite bus in the same analysis. The equivalent adjusted Pi-line representation* per Figure 2 has been made by separate calculation.

The list of symbols and Fortran equivalent names are given in the Appendix A. The actual per-unit, (pu) values on the machine base used are found in Programs 1 and 2 of the Appendix C. It should be noted that since power factor is the independent variable and with generator power output held constant, pu current equals pu power output divided by the product of power factor times the pu terminal voltage. Rated three-phase mva and rated line-to-line terminal voltage are the two machine bases, both taken as 1.0 pu. Rated full-load power in pu is then 0.9 pu for a 0.90 power factor machine and must be so read into the program. The infinite bus voltage, V_b, is held at 1.0 pu as long as the terminal voltage, V_t, does not fall below 0.8 pu to maintain this condition. V-terminal must be determined by iterating with each change in power factor.

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*Westinghouse Electric Corporation (33, p. 44).

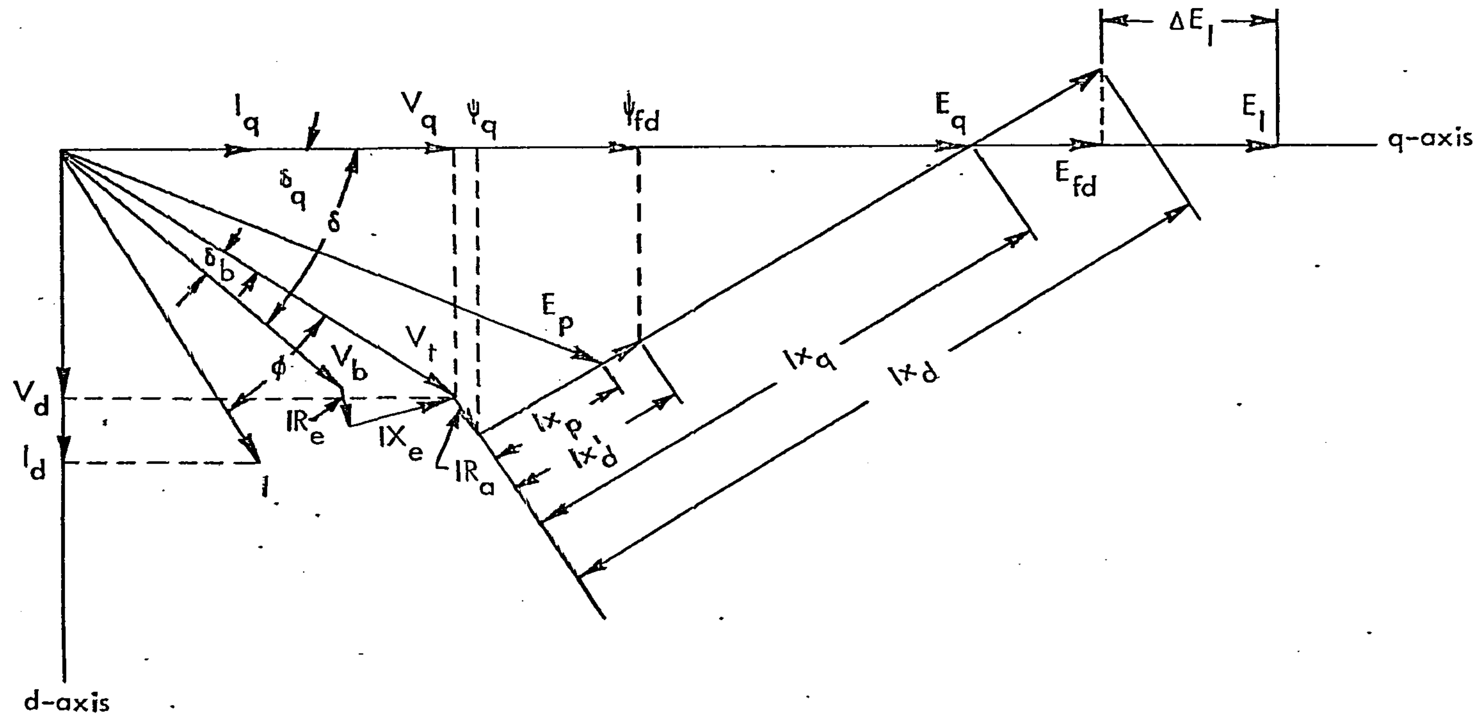


Figure 1. Phasor diagram of generator and tie-line

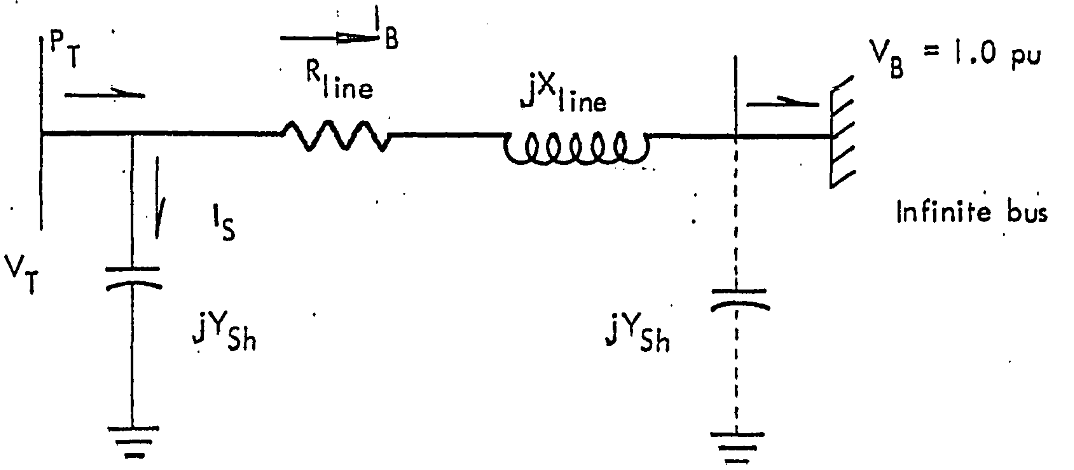


Figure 2. Model of transmission line for the calculation of the terminal voltage of the generator

It should be noted that the tie-line or transmission line inductive reactance often identified by x-line rather than x_e refers to its equivalent pu reactance on the machine base.

The effect of change of speed on generated voltages due to the damper winding has been assumed as negligible in this analysis. Balanced loads are assumed.

A. Operating Matrix Equations

When a Taylor-series expansion is formed about any operating point, for each of the equations (54-589), the resulting linear perturbation relationships of the first order may be expressed as in Equation 1. This matrix equation summarizes the relationship between all machine and system variables where the inputs are ΔT_m through the prime mover and Δv_{fd} through the excitation system. The controlled machine variables $\Delta\delta$, ΔV_t and ΔI, and all time-derivative quantities are in the first two equations. (See following page for Equation 1.)

B. Reduced System A-constants

Equation 1 has the matrix form.

$$\underline{F} = \underline{M}\ \underline{G} \qquad (2)$$

or, in the expanded form.

$$
\begin{array}{c}
1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11
\end{array}
\left[\begin{array}{c}
\Delta T_m\\
\Delta v_{fd}\\
\\
\\
\hline
\\
\\
\\
\\
\\
\\
\\
\end{array}\right]
=
\left[\begin{array}{cccc|ccccccc}
Mp^2 + Dp & & & & & & & -\psi_q & \psi_d & i_q & -i_d\\
 & & & & & & (1+\tau_f p) & -\tau_f p(x_d - x_d') & & & \\
 & & 1 & & \frac{-v_d}{V_t} & \frac{-v_q}{V_t} & & & & & \\
 & & & 1 & & & & \frac{-i_d}{I} & \frac{-i_q}{I} & & \\
\hline
-V_b\cos\delta & & & & 1 & & & -r_e & x_e & & \\
V_b\sin\delta & & & & & 1 & & -x_e & -r_e & & \\
 & & & & & 1 & -1 & x_d & r_q & & \\
 & & & & 1 & & & r_d & -x_q & & \\
 & & & & & & & & -x_q & & 1\\
 & & & & & & -1 & x_d & & 1 & \\
 & 1 & & & & & -1 & (x_d - x_d') & & &
\end{array}\right]
\cdot
\left[\begin{array}{c}
\Delta\delta\\
\Delta\psi_{fd}\\
\Delta V_t\\
\Delta I\\
\hline
\Delta v_d\\
\Delta v_q\\
\Delta i_{fd}\\
\Delta i_d\\
\Delta i_q\\
\Delta\psi_d\\
\Delta\psi_q
\end{array}\right]
\qquad (1)
$$

$$\begin{bmatrix} \Delta T_m \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} M_1 & M_2 \\ \hline M_3 & M_4 \end{array}\right] \cdot \begin{bmatrix} \Delta\delta \\ \vdots \\ \Delta I \\ \hline \Delta v_d \\ \vdots \\ \Delta\psi_q \end{bmatrix} \tag{3}$$

and therefore the variables not of interest, Δv_d to $\Delta\psi_q$, may be eliminated by matrix reduction, yielding

$$\begin{bmatrix} \Delta T_m \\ \Delta v_{fd} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} M_1 - M_2 M_4^{-1} M_3 \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta \\ \Delta\psi_{fd} \\ \Delta V_t \\ \Delta I \end{bmatrix} \tag{4}$$

Writing, for purposes of brevity, $R_d = r_d + r_e$, $X_d = x_d + x_e$ and continuing for R_q, X_q, X_d', Equation 4 can be expanded as

$$\begin{bmatrix} \Delta T_m \\ \Delta v_{fd} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Mp^2 + Dp - A_1 & -A_2 & 0 & 0 \\ -A_3 & -A_4(1+\tau_{dz}'p) & 0 & 0 \\ -A_5 & -A_6 & 1 & 0 \\ -A_7 & -A_8 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta \\ \Delta\psi_{fd} \\ \Delta V_t \\ \Delta I \end{bmatrix} \tag{5}$$

where the relationships summarized by the constants A_1 - A_8, are given in Appendix B. These are the A-constants found in Laughton (21) which also are directly related to constants formed by Heffron and Phillips (14). Laughton found that eliminating resistances introduced a five percent error. With a fast computer the added correctness warranted retaining all resistances.

The matrix reduction was done algebraically to confirm Laughton's A-constants. It can be done by digital matrix reduction as well, switching rows so that the principal diagonal contains no zeros.

C. Feedback Control Scheme

The operating matrix Equation 1 and the reduced system matrix Equation 5 represent an open-loop system of one machine connected to an equivalent infinite bus system with two arbitrary forcing functions, the excitation system and the prime mover. Addition of further regulating equipment may be made with the field control responding to changes in terminal voltage, current and rotor angle. Additional equations, including the governor are

$$\Delta v_{fd} = -G_1(p)\{\Delta V_t + G_2(p)\Delta I + G_3(p)\Delta\delta\} \tag{6}$$

and

$$\Delta T_m = -G_4(p)\Delta\delta \tag{7}$$

where $G_1(p)$, $\ldots G_4(p)$ are the transfer functions of these respective sub-systems.

The block diagram of the above control scheme and machine with feed-back loops is given in Figure 3.

D. State-space Equations

It is now possible to rearrange the Equation 5 by state-space methods (21) to the required set of first-order equations. For linear constant-coefficient systems, the equations take the form

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{D}\,\underline{u} \tag{8}$$

A first-order vector differential equation, where $\underline{A}$ and $\underline{D}$ are matrixes of constant coefficients, $\underline{u}$ is the forcing-function inputs and $\underline{x}$ the vector representing the state of the system variables.

Letting

$$x_1 = \Delta\delta$$

$$x_2 = \dot{x}_1 = \Delta\dot{\delta}$$

$$x_3 = \Delta\psi_{fd}$$

and control input variables,

$$u_1 = \Delta v_{fd}$$

$$u_2 = \Delta T_m\,,$$

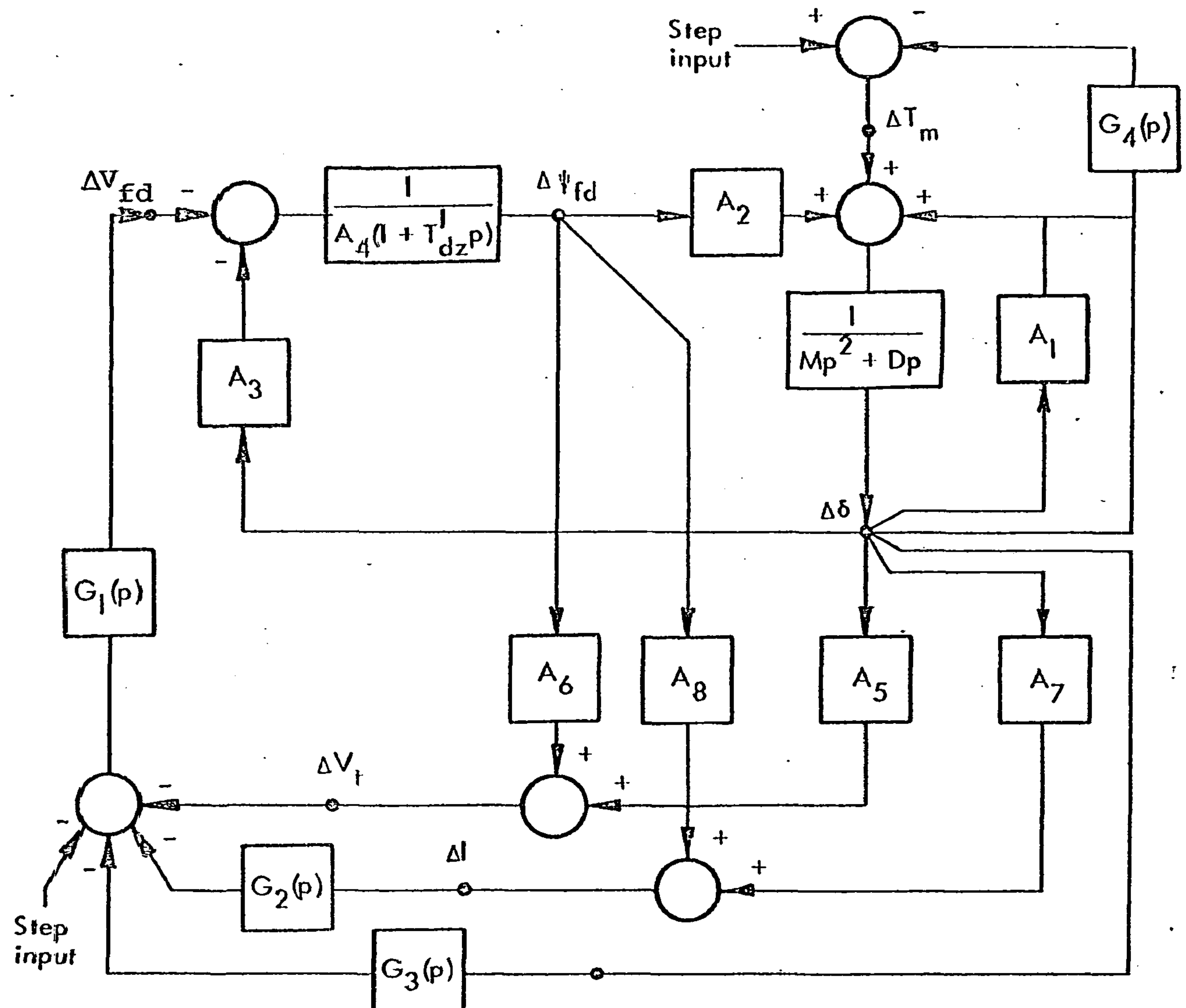


Figure 3. Block diagram of synchronous generator connected to an equivalent power system with multi-loop machine feedback control system

Equation 5 becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{A_1}{M} & \frac{-D}{M} & \frac{A_2}{M} \\ \frac{-A_3}{A_4\tau'_{dz}} & 0 & \frac{-1}{\tau'_{dz}} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M} \\ \frac{-1}{A_4\tau'_{dz}} & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (9)$$

The output may also be represented by

$$\underline{y} = \underline{B}\,\underline{x} \quad (10)$$

in which case Equation 5 can be expanded by substituting $y_1 = \Delta V_t$, $y_2 = \Delta I$ giving

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} A_5 & 0 & A_6 \\ A_7 & 0 & A_8 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (11)$$

The stability of the system by Equation 5 is assured when the forcing functions $\underline{u}$ are finite values, only if all the eigenvalues of the coefficient matrix $\underline{A}$ as found from

$$\mathrm{Det}(\underline{A} - \lambda\underline{I}) = 0 \quad (12)$$

have negative real parts.

E. Damping

The system damping including that of the damper winding circuit is investigated at two values, D = 2 and D = 3. It does not include the effect of rotor damping currents on the generated voltage.

Concordia (6) states that except for the case of cross-compound generator sets, omitting the damper winding (or amortisseur winding) has little effect on steady-state stability. Likewise Laughton (21) states damping is of no great importance as to locating the stability limit.

Undrill (30) treats the damper winding separately with speed voltage effects. The necessity and accuracy of this addition is an area for separate study.

F. Saturation

Generator saturation manifests its effect in two ways, in the transient reactance and in the excitation requirements.

1. Reactances

Kilgore (18) has published the effect of saturation in depth. Based on Kilgore's data the transient reactance used for x'_d was taken at 0.93 times the saturated transient reactance at 1.0 pu current. All other values of reactances and open-circuit field time constants were based on unsaturated conditions.

2. Excitation requirements

The generator saturation effect is taken from Young and Webler (34). The equation for the increase in excitation voltage, ΔE_I per Figure 1 is given by

$$\Delta E_I = A_G(\varepsilon^{B_G(E_p - 0.8)}) \tag{13}$$

where E_p is the voltage behind Potier reactance x_p and A_G and B_G are constants derived from taking two points on the saturation curve, Figure 4.

A_G and B_G, calculated separately and found to be 0.035 and 7.358 respectively, appear in this form in Program 1 of Appendix C. E_p is calculated as $E_p = V_T + Ix_p$ in complex form. Finally ΔE_I is added to E_{fd} giving the saturated or actual pu field voltage needed.

Generator field heating is a possible limitation on over-excited operation with the infinite bus held at 1.0 pu voltage.

G. Tie-line Reactance

The tie-line or transmission line is an equivalent Pi-representation given by Figure 2. The bus voltage is held at 1.0 pu of generator terminal voltage as long as the generator terminal voltage does not fall below 0.8 pu. By iteration, in Program 1, Appendix C, the terminal voltage is adjusted to give V_b = 1.0 pu, without tap-change, before

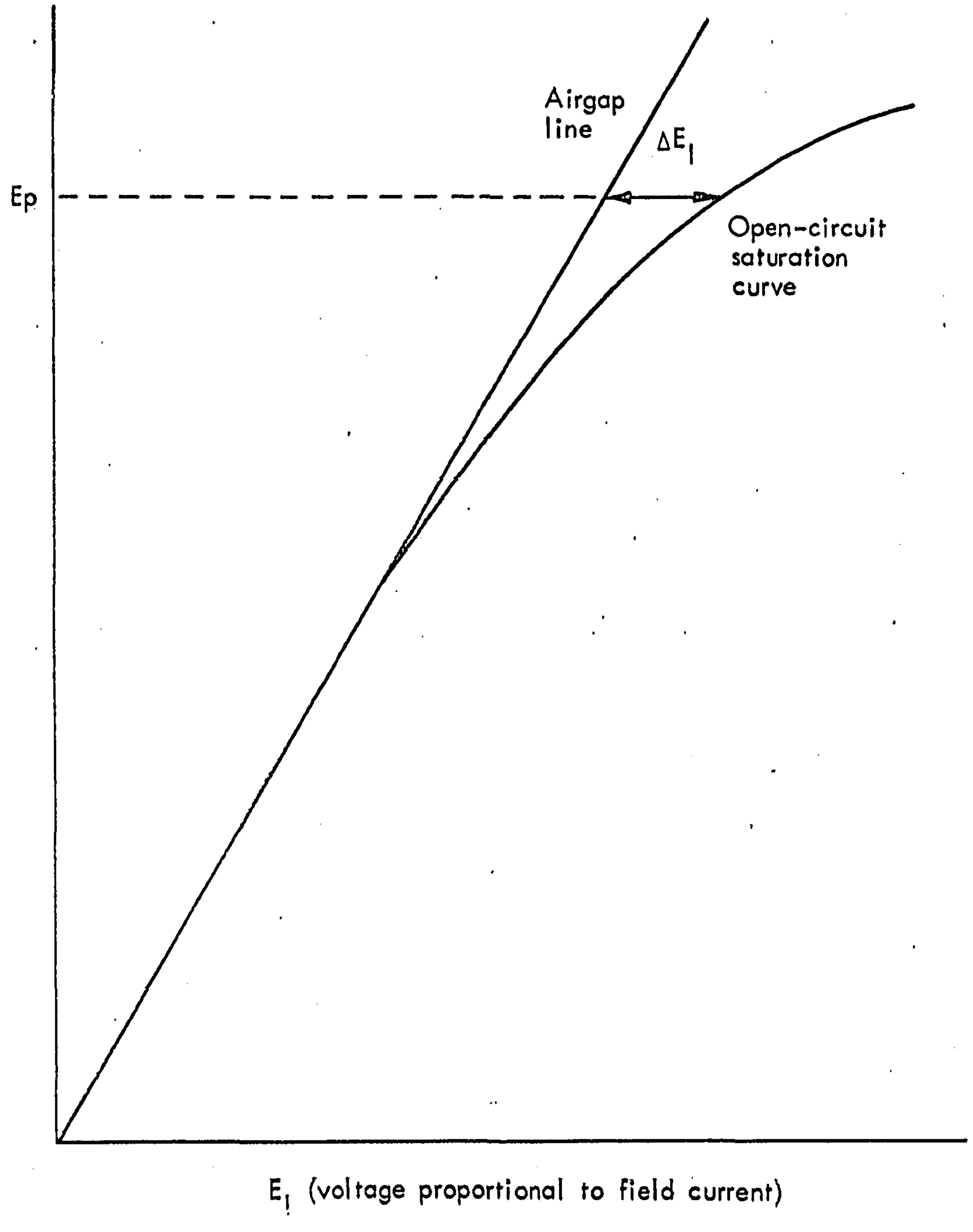


Figure 4. Representation of saturation of generator

further calculations are made.

Two values of tie-line impedance were investigated. Since the value of the tie-line reactance has a great effect on stability, tabulations are all identified by the tie-line reactance used, namely, X-LINE = 0.414 pu or X-LINE = 0.718 pu. These are Pi-line adjusted values. Programs 1 and 2 of Appendix C input values that are associated with the lower and higher tie-line reactances respectively. The use of shunt capacitors could easily be added as a separate study.

V. EXCITATION SYSTEM

The Institute of Electrical and Electronics Engineers power generating excitation systems subcommittee has formulated and reported recommended computer representations for four major types of excitation systems (16). The constants and symbols used with block diagrams, per unit notation, and exciter saturation representation used below follow this committee report.

A. Type 1 System

Figure 5 is the block diagram of the excitation system. One of the signals initially summed is the time derivative of electric power. The state-space equations for the type 1 system will be developed and then the power signal treated separately.

Following Anderson and Fouad (1) the type 1 system may be redrawn as Figure 6.

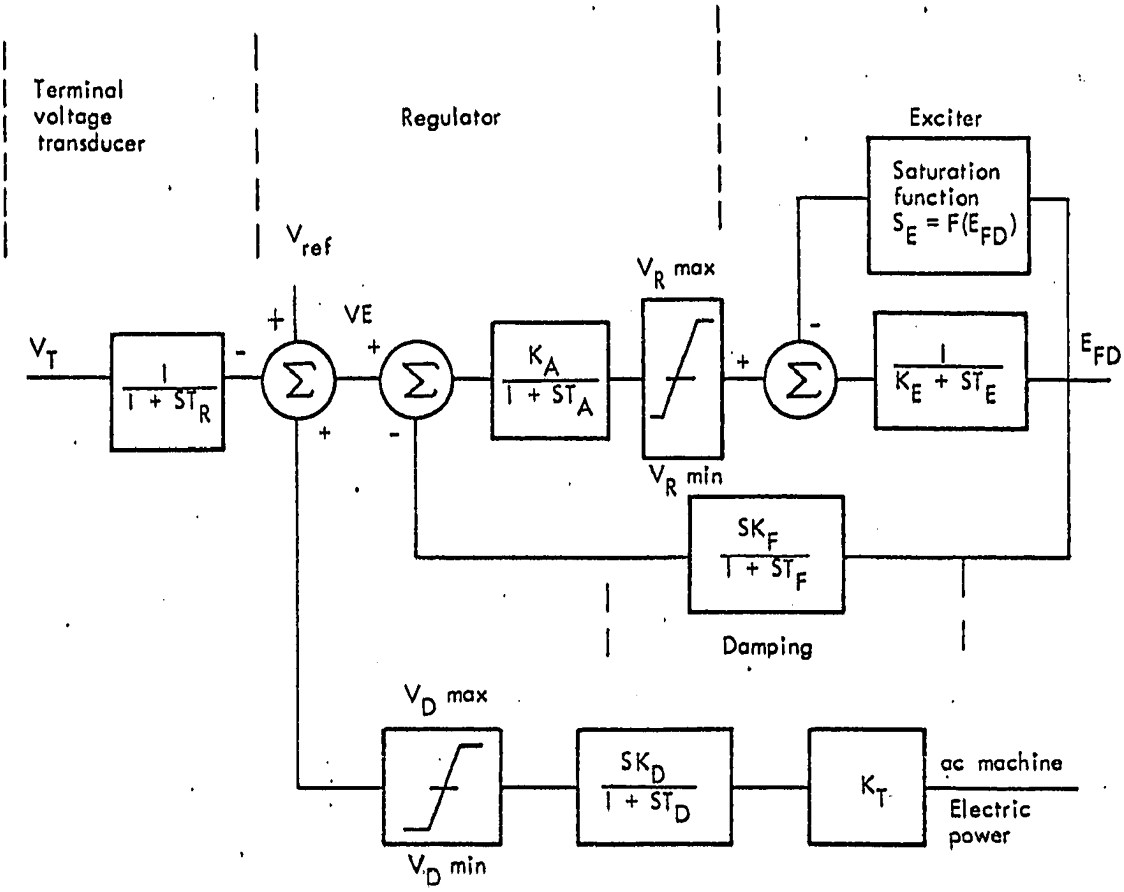
Now

$$\frac{V_F}{V_E} = \frac{G_2G_3}{1 + G_3H_1 + G_2G_3H_2} \tag{14}$$

where

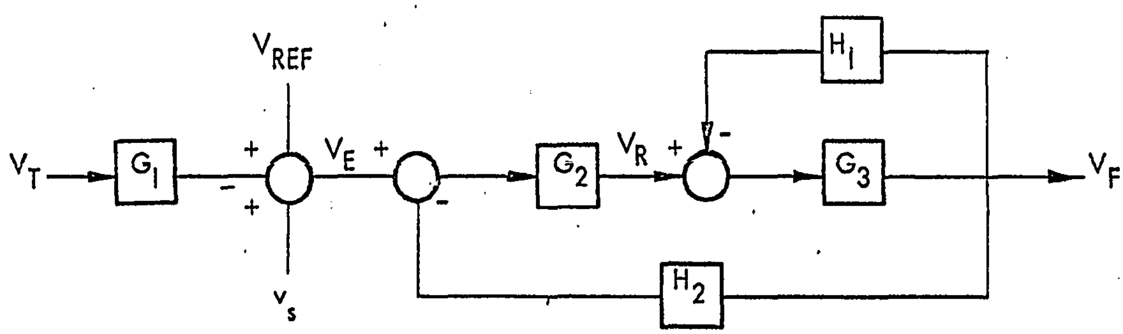
$$V_E = V_{REF} + V_S - G_1V_T$$

Substituting and rearranging



E_{FD}	Exciter output voltage (applied to generator field)
K_A	Regulator gain
K_E	Exciter constant related to self-excited field
K_F	Regulator stabilizing circuit gain
T_A	Regulator amplifier time constant
T_E	Exciter time constant
T_F	Regulator stabilizing circuit time constant
T_R	Regulator input filter time constant
V_R	Regulator output voltage
V_T	Generator terminal voltage
K_D	Damping gain
T_D	Damping time constant
VE	Intermediate variable point

Figure 5. Type 1 excitation system block diagram



Where $G_1 = \frac{1}{1 + ST_R}$ $\qquad H_1 = S_E$

$G_2 = \frac{K_A}{1 + ST_A}$ $\qquad H_2 = \frac{SK_F}{1 + ST_F}$

$G_3 = \frac{1}{K_E + ST_E}$

Figure 6. Simplified block diagram of type 1 excitation system

$$\frac{V_F}{V_E} = \frac{K_A(1+ST_F)}{(1+ST_A)(K_E+ST_E)(1+ST_F) + S_E(1+ST_A)(1+ST_F) + K_AK_FS} \tag{15}$$

The right-hand side denominator, call it $\phi(s)$, is the characteristic equation. Rearranging

$$\phi(s)V_F = K_A(1+ST_F)V_E \tag{16}$$

where

$$\begin{aligned}\phi(s) = {} & T_AT_ET_FS^3 + [K_ET_AT_F + T_E(T_A+T_F) + S_ET_AT_F]S^2 \\ & + [(K_E+S_E)(T_A+T_F) + T_E + K_AK_F]S + (K_E+S_E)\end{aligned} \tag{17}$$

dividing by $(T_AT_ET_F)$ and taking the inverse Laplace transform gives

$$\dddot{v}_f + a_1\ddot{v}_f + a_2\dot{v}_f + a_3v_f = b_o\dddot{v}_e + b_1\ddot{v}_e + b_2\dot{v}_e + b_3v_e$$

where

$$a_1 = \frac{T_E(T_A+T_F) + T_AT_F(K_E+S_E)}{T_AT_ET_F}$$

$$a_2 = \frac{(K_E+S_E)(T_A+T_F) + T_E + K_AK_F}{T_AT_ET_F}$$

$$a_3 = \frac{K_E + S_E}{T_AT_ET_F} \tag{18}$$

$$b_o = b_1 = 0$$

$$b_2 = \frac{K_A}{T_A T_E}$$

$$b_3 = \frac{K_A}{T_A T_E T_F}$$

Rewriting Equation 18 terms of p-operator as

$$(p^3 + a_1 p^2 + a_2 p + a_3)\, v_f =$$

$$(b_o p^3 + b_1 p^2 + b_2 p + b_3)\, v_e$$

or

$$p^3(v_f - b_o v_e) + p^2(a_1 v_f - b_1 v_e) + p(a_2 v_f - b_2 v_e)$$

$$+ (a_3 v_f - b_3 v_e) = 0 \tag{19}$$

and rearranging after dividing by p^3 we get

$$v_f - b_o v_e = \underbrace{\frac{1}{p}(b_1 v_e - a_1 v_f) + \frac{1}{p}\underbrace{\left[\frac{1}{p}(b_2 v_e - a_2 v_f) + \frac{1}{p}\underbrace{\left(\frac{1}{p}(b_3 v_e - a_3 v_f)\right)}_{x_6}\right]}_{x_5}}_{x_4} \tag{20}$$

calling $x_4 = v_f - b_o v_e$, following the underscoring brackets

$$x_5 = \dot{x}_4 + (a_1 v_f - b_1 v_e)$$

$$x_6 = \dot{x}_5 + (a_2 v_f - b_2 v_e) \tag{21}$$

To place the equations in state-space form $\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{D}\,\underline{u}$, noting that $v_f = x_4 + b_o v_e$ now

$$\begin{aligned} x_6 &\equiv \frac{1}{p}(b_3 v_e - a_3 v_f) \\ p x_6 &= b_3 v_e - a_3 v_f \\ \dot{x}_6 &= b_3 v_e - a_3(x_4 + b_o v_e) \\ \dot{x}_6 &= -a_3 x_4 + (b_3 - a_3 b_o) v_e \end{aligned} \tag{22}$$

and

$$\begin{aligned} x_5 &\equiv \frac{1}{p}(b_2 v_e - a_2 v_f + x_6) \\ \dot{x}_5 &= b_2 v_e - a_2(x_4 + b_o v_e) + x_6 \\ \dot{x}_5 &= -a_2 x_4 + x_6 + (b_2 - a_2 b_o) v_e \end{aligned} \tag{23}$$

and

$$\begin{aligned} x_4 &\equiv \frac{1}{p}(b_1 v_e - a_1 v_f) + x_5 \\ \dot{x}_4 &= b_1 v_e - a_1(x_1 + b_o v_e) + x_5 \\ \dot{x}_4 &= -a_1 x_4 + x_5 + (b_1 - a_1 b_o) v_e \end{aligned} \tag{24}$$

In matrix form $\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{D}\,\underline{u}$, noting $b_o = b_1 = 0$

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix} \cdot + \begin{bmatrix} b_1 - a_1 b_o \\ b_2 - a_2 b_o \\ b_3 - a_3 b_o \end{bmatrix} \cdot \begin{bmatrix} v_e \end{bmatrix} \tag{25}$$

Addition of the three "inputs"

$$V_E = \frac{-V_T}{1 + ST_R} + V_{REF} + V_S$$

from which

$$\dot{v}_e + \frac{v_e}{T_R} = -V_T + \frac{v_{ref} + v_s}{T_R}$$

calling $x_7 = v_e$ this would change the matrix to

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 & (b_1 - a_1 b_o) \\ -a_2 & 0 & 1 & (b_2 - a_2 b_o) \\ -a_3 & 0 & 0 & (b_3 - a_3 b_o) \\ 0 & 0 & 0 & \frac{-1}{T_R} \end{bmatrix} \cdot \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-1}{T_R} \end{bmatrix} \cdot \begin{bmatrix} v \end{bmatrix} \tag{26}$$

where $v = -v_t + v_{ref} + v_s$.

B. Power Damping Signal

The lower blocks in Figure 5 inputs a damping power signal. The time constant T_R is zero. This alters as follows:

$$V_E = -V_T + V_{REF} + \frac{K_T S K_D P}{1 + ST_D}$$

from which

$$v_e + T_D \dot{v}_e = -v_t + v_{ref} + K_T K_D \frac{dP}{d\delta} \cdot \frac{d\delta}{dt}$$

and

$$\dot{v}_e = \frac{-v_e - v_t + v_{ref}}{T_D} + \frac{K_T K_D}{T_D} \frac{dP}{d\delta} \cdot \frac{d\delta}{dt} \tag{27}$$

Now

$$\frac{d\delta}{dt} = x_2 = \dot{x}_1 = \Delta\dot{\delta}$$

from the machine state-space equation and $\frac{dP}{d\delta}$ can be calculated from Concordia*

$$P = \frac{EV_T}{x_d'} \sin \delta + \left(\frac{1}{x_q} - \frac{1}{x_d'}\right) \frac{V_T^2}{2} \sin 2\delta$$

and

$$\frac{dP}{d\delta} = \frac{EV_T \cos \delta}{x_d'} + \left(\frac{1}{x_q} - \frac{1}{x_d'}\right) V_T^2 \cos 2\delta \tag{28}$$

---

*Concordia (8, p. 38).

where

$$E = \psi_d + x'_d i_d.$$

The regulator can now be combined with the machine. K_A inputs the signals to excitation system level similar to Laughton's* G_1 function. In this machine a generator field voltage of 274 volts produces 1.0 per unit generator volts on the air-gap line. This is the K_2 scaling factor, per Laughton.*

C. Saturation

Figure 7 shows the meaning of the saturation function and how it is actually calculated. In the same manner as Equation 13 was used to calculate ΔE_I for the generator saturation, in a like manner S_E can be calculated from two values from the exciter saturation curve or equivalent data.

$$S_E = A_E(\varepsilon^{B_E(E_{FDS}-.8)}) \tag{29}$$

In this exciter A_E was calculated to be 0.235 and B_E was calculated to be 0.214. S_E was limited to give a maximum of 0.835 in the program for cases of overexcited operation.

---

*Laughton (21, p. 335).

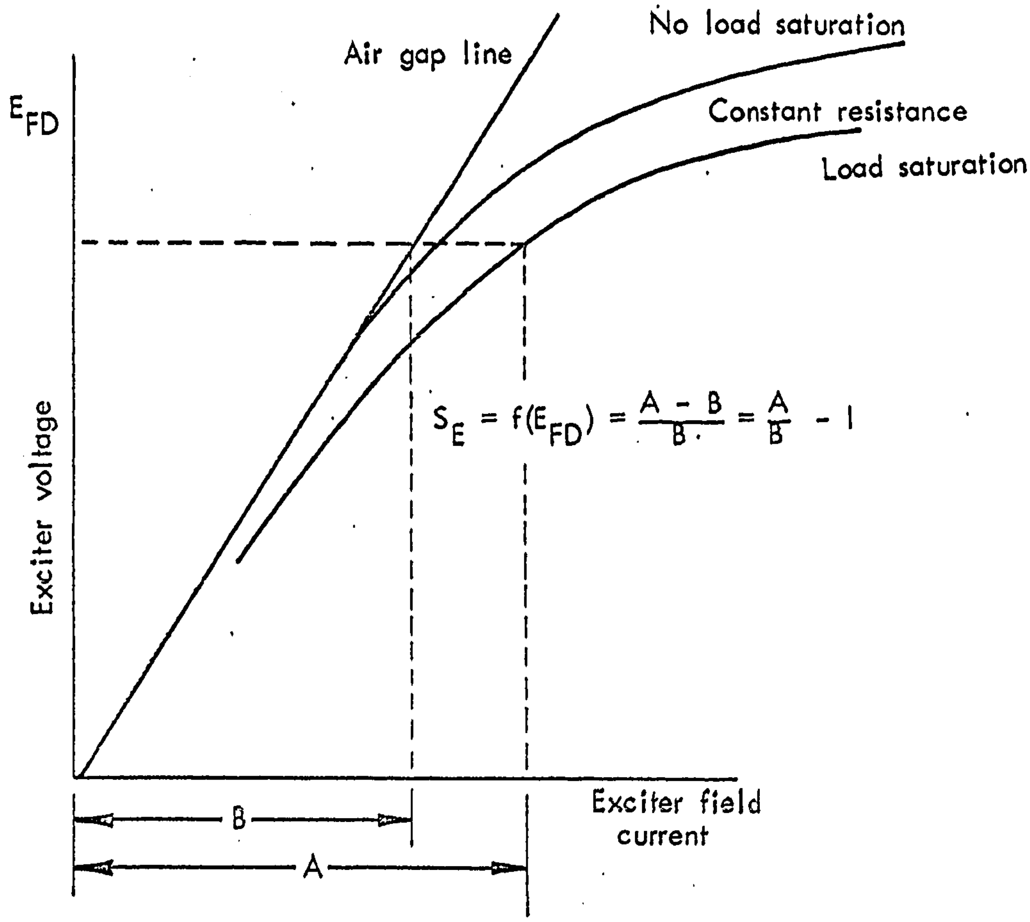


Figure 7. Calculation of exciter saturation

D. State-space Equations

Equation 30 gives the resulting state-space matrix formulation. Note $b_o = b_1 = 0$.

$$
[\dot{x}] = \begin{array}{c}
\underline{A} - \text{matrix} \\
\left[\begin{array}{c|c|c|c|c|c|c}
 & 1 & & & & & \\
\hline
\frac{A_1}{M} & \frac{-D}{M} & \frac{A_2}{M} & & & & \\
\hline
\frac{-A_3}{A_4 T'_{dz}} & & \frac{-1}{T'_{dz}} & \frac{-K_2}{A_4 T'_{dz}} & & & \\
\hline
 & & & -a_1 & 1 & & \\
\hline
 & & & -a_2 & & 1 & b_2 \\
\hline
 & & & -a_3 & & & b_3 \\
\hline
 & \frac{K_T K_D}{T_D}\frac{dP}{d\delta} & & & & & \frac{-1}{T_D}
\end{array}\right]
\end{array}
\begin{array}{c}
\underline{x} \\
\left[\begin{array}{c}
x_1 = \Delta\delta \\
\hline
x_2 = \Delta\dot{\delta} \\
\hline
x_3 = \Delta\psi_{fd} \\
\hline
x_4 = \Delta V_f \\
\hline
x_5 \\
\hline
x_6 \\
\hline
x_7 = \Delta V_e
\end{array}\right]
\end{array}
$$

$$
+ \begin{array}{c}
D \\
\left[\begin{array}{c|c}
\frac{1}{M} & \\
\hline
 & \\
\hline
 & \\
\hline
 & \\
\hline
 & \\
\hline
 & \\
\hline
 & \frac{-1}{T_D}
\end{array}\right]
\end{array}
\cdot
\begin{array}{c}
\underline{U} \\
\left[\begin{array}{c}
\Delta T_M \\
\Delta V
\end{array}\right]
\end{array}
\quad \text{Where } V = -\Delta V_t + \Delta V_{ref} \qquad (30)
$$

x_4, x_5, x_6, x_7 derive from state formulation of type I excitation system.

VI. GOVERNOR SYSTEM

The single-shaft steam turbine driving the generator is given by block diagram representation in Figure 8 which includes the details of the prime mover and governor. State-space equations are formulated. The addition of an auxiliary control signal, tie-line control, is treated separately. Gain K_1 has been divided by rated mw to place it in per-unit level.

A. Single-shaft Governor Representation

State-space variable approach from Anderson and Fouad (1) will be followed. The steam system dynamics includes steam and reheat time constants and the fraction of reheat. The governor includes water hammer correction, the servo valve motor, and control system time constants. The block diagram of the single-shaft governor is given in Figure 8.

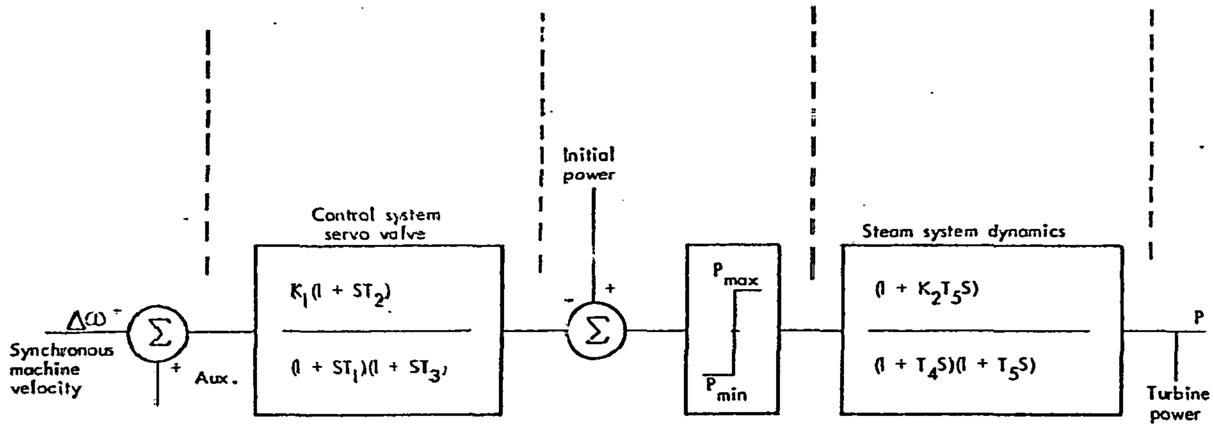
B. State-space Equations

1. Steam-system dynamics

The transfer function of the steam system Figure 9 by parallel programming is:

$$G(s) = \frac{1 + K_2 T_5 S}{(1 + T_4 S)(1 + T_5 S)} \qquad (31)$$

By partial fraction expansion



T_1 Control system time constant
T_2 Water hammer correction
T_3 Servo valve motor
T_4 Steam system time constant
T_5 Reheat Steam system time constant
K_1 Overall gain (per unit power/radian/sec.)
K_2 Fraction of steam reheated

Figure 8. Block diagram of governor system

$$G(s) = \frac{A}{1 + T_4 S} + \frac{B}{1 + T_5 S} \tag{32}$$

Solving for A and B

$$A = \frac{T_4 - K_2 T_5}{T_4 - T_5}$$

$$B = \frac{T_5(1 - K_2)}{T_5 - T_4}$$

and substituting in Equation 32 gives

$$G(s) = \frac{(T_4 - K_2 T_5)/T_4(T_4 - T_5)}{S + 1/T_4} + \frac{(1 - K_2)/(T_5 - T_4)}{S + 1/T_5} \tag{33}$$

calling

$$A' = (T_4 - K_2 T_5)/T_4(T_4 - T_5)$$

$$B' = (1 + K_2)/(T_5 - T_4)$$

the branches take the forms

$$\frac{u}{x} = \frac{A'}{s + 1/T_4}$$

and

$$\frac{u}{x} = \frac{B'}{s + 1/T_5}$$

Combining as in Figure 10 the state-space variables are:

$$\dot{x}_8 = \frac{-1}{T_4} x_8 + u$$

$$\dot{x}_9 = \frac{-1}{T_5} x_9 + u$$

and

$$x = A' x_8 + B' x_9 \tag{34}$$

2. Governor control system and servo valve

The transfer function of the governor itself, Figure 8 by parallel programming is:

$$G(s) = \frac{K_1(1 + T_2 S)}{(1 + T_1 S)(1 + T_3 S)} \tag{35}$$

By partial fraction expansion

$$G(s) = \frac{B_1}{1 + T_1 S} + \frac{B_2}{1 + T_3 S} \tag{36}$$

Solving for B_1 and B_2

$$B_1 = \frac{K_1(T_1 - T_2)}{(T_1 - T_3)}$$

$$B_2 = \frac{K_1(T_3 - T_2)}{(T_3 - T_1)}$$

Substituting in Equation 36 gives

$$G(s) = \frac{K_1(T_1 - T_2)/T_1(T_1 - T_3)}{S + 1/T_1} + \frac{K_1(T_3 - T_2)/T_3(T_3 - T_1)}{S + 1/T_3} \tag{37}$$

calling

$$B_1' = K_1(T_1 - T_2)/T_1(T_1 - T_3)$$

$$B_2' = K_1(T_3 - T_2)/T_3(T_3 - T_1)$$

Combining as per Figure 11 the state-space variables are

$$\dot{x}_{10} = -\frac{1}{T} x_{10} + u_{in}$$

$$\dot{x}_{11} = -\frac{1}{T_3} x_{11} + u_{in}$$

$$u' = B_1' x_{10} + B_2' x_{11} \tag{38}$$

Combining the two transfer functions per Figure 12

$$u_{in} = -\Delta\dot{\delta} + \omega_s = \omega_s - x_2 \tag{39}$$

where $x_2 = \Delta\dot{\delta}$; ω_s = auxiliary signal. Substituting in Equation

$$\dot{x}_{10} = -\frac{1}{T_1} x_{10} - x_2 + \omega_s$$

$$\dot{x}_{11} = \frac{1}{T_3} x_{11} - x_2 + \omega_s$$

Solving for $\dot{x}_8$ and $\dot{x}_9$

$$u_\Delta = P_{\Delta o} - u'_\Delta$$

$$= P_{\Delta o} - B_1' x_{10} - B_2' x_{11} \tag{40}$$

where subscript Δ refers to a small increment of that variable

$$\dot{x}_8 = -\frac{1}{T_4} x_8 + u_\Delta$$

$$= -\frac{1}{T_4} x_8 - B_1' x_{10} - B_2' x_{11} + P_{\Delta o}$$

and

$$\dot{x}_9 = -\frac{1}{T_5} x_9 - B_1' x_{10} - B_2' x_{11} + P_{\Delta o} \qquad (41)$$

State-space equations for governor system are

$$\dot{x}_8 = -\frac{1}{T_4} x_8 - B_1' x_{10} - B_2' x_{11} + P_{\Delta o}$$

$$\dot{x}_9 = -\frac{1}{T_5} x_9 - B_1' x_{10} - B_2' x_{11} + P_{\Delta o}$$

$$\qquad (42)$$

$$\dot{x}_{10} = -\frac{1}{T_1} x_{10} - x_2 + \omega_s$$

$$\dot{x}_{11} = -\frac{1}{T_3} x_{11} - x_2 + \omega_s$$

The final combined state-space formulation of generator, excitation system and governor control system is given in Equation 43.

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \\ \dot{x}_{11} \end{bmatrix}
=
\begin{array}{c|ccccccccccc|}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
\hline
1 & & 1 & & & & & & & & & \\
2 & \frac{A_1}{M} & \frac{-D}{M} & \frac{A_2}{M} & & & & & & & & \\
3 & \frac{-A_3}{A_4 T' dz} & & \frac{-1}{T' dz} & \frac{-K_2}{A_4 T' dz} & & & & & & & \\
4 & & & & -a_1 & 1 & & & & & & \\
5 & & & & -a_2 & & 1 & b_2 & & & & \\
6 & & & & -a_3 & & & b_3 & & & & \\
7 & \frac{-A_5}{T_D} & * & \frac{-A_6}{T_D} & & & & -\frac{1}{T_D} & & & & \\
8 & & & & & & & & -\frac{1}{T_4} & & -B_1' & -B_2' \\
9 & & & & & & & & & -\frac{1}{T_5} & -B_1' & -B_2' \\
10 & & -1 & & & & & & & & -\frac{1}{T_1} & \\
11 & & -1 & & & & & & & & & -\frac{1}{T_3} \\
\hline
\end{array}
\cdot
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \end{bmatrix}
+
$$

$* \; \frac{K_T K_D}{T_D} \; \frac{dP}{d\delta}$

$$
\begin{array}{c|c|}
\hline
1 & \\
2 & \frac{1}{M} \\
3 & \\
4 & \\
5 & \\
6 & \\
7 & \\
8 & \\
9 & \\
10 & \\
11 & \\
\hline
\end{array}
\cdot \boxed{\Delta T_M}
+
\begin{array}{c|c|}
\hline
1 & \\
2 & \\
3 & \\
4 & \\
5 & \\
6 & \\
7 & \\
8 & 1 \\
9 & 1 \\
10 & \\
11 & \\
\hline
\end{array}
\cdot \boxed{\Delta P_0}
+
\begin{array}{c|cc|}
\hline
1 & & \\
2 & & \\
3 & & \\
4 & & \\
5 & & \\
6 & & \\
7 & & \frac{-1}{T_D} \\
8 & & \\
9 & & \\
10 & 1 & \\
11 & 1 & \\
\hline
\end{array}
\cdot
\begin{bmatrix} \Delta\omega_s \\ \Delta V_s \end{bmatrix}
\qquad (43)
$$

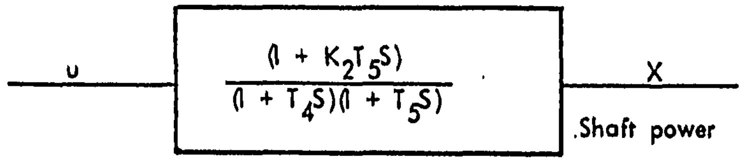


Figure 9. Transfer function of steam system

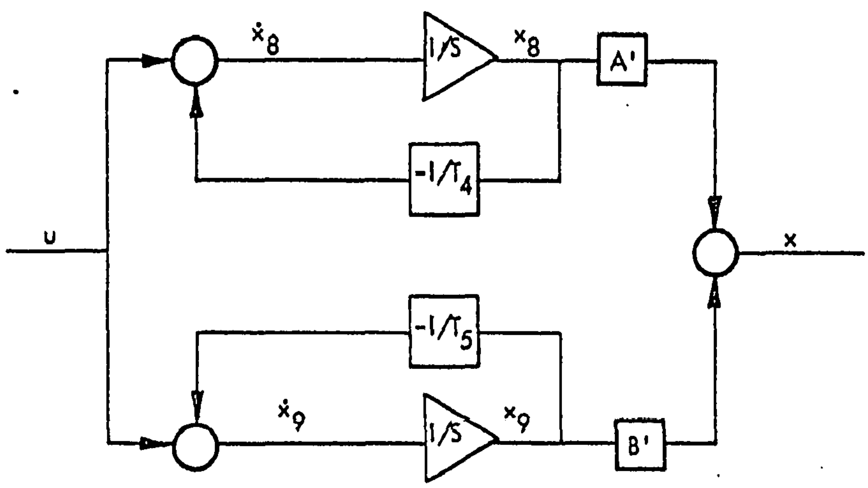


Figure 10. State-space diagram of steam system

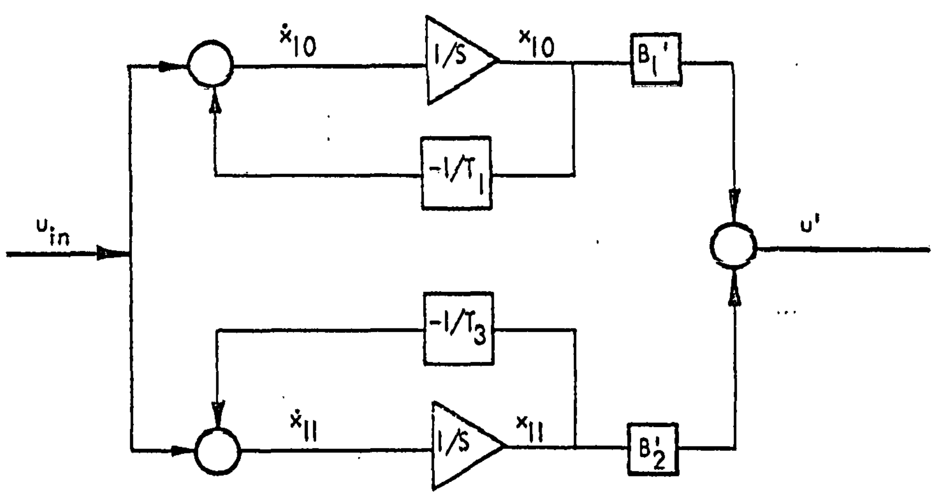


Figure 11. State-space diagram of control system and servo valve

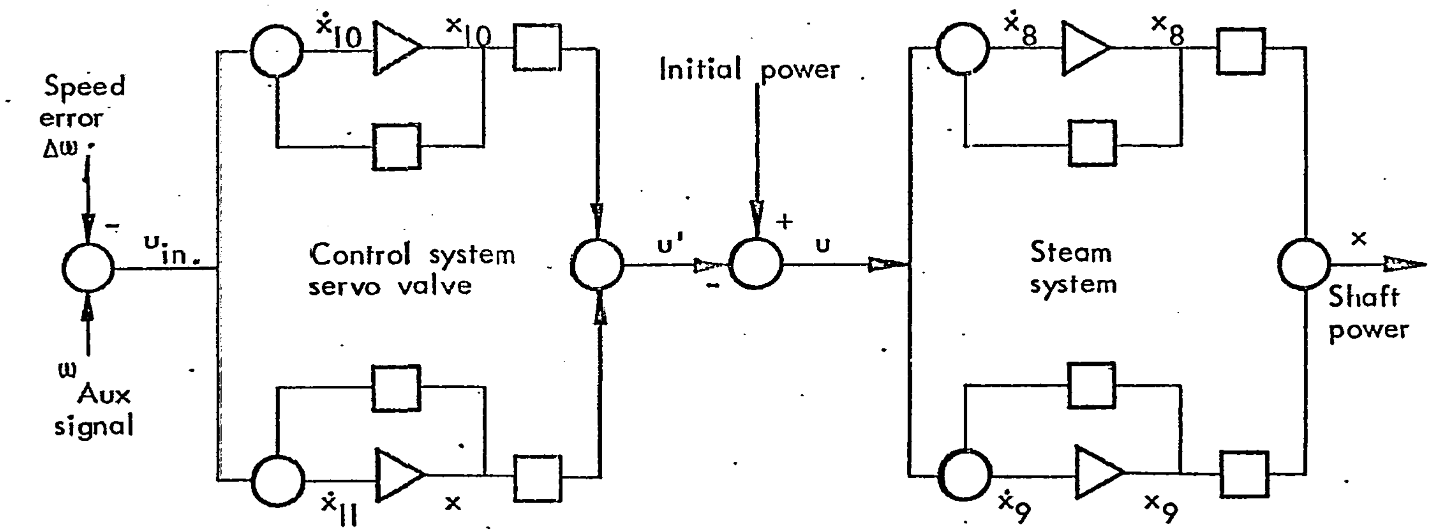


Figure 12. State space diagram of governor system

VII. TIE-LINE CONTROLLER

For area control, a tie-line controller can be added. The following method by VanNess gives the desired additional equations.

A. Tie-line Controller Representation

Tie-line, load frequency, control signals can be fed into the governor input as auxiliary signals, see Figure 13. VanNess' (31) speed-droop characteristic 1/R is separated from K_1 gain in the previous discussion. P_C is the reset control signal.

B. State-space Equations

The additions to the previous matrix equations by the controller are given by

$$\dot{X}_R = K_R X_P \tag{44}$$

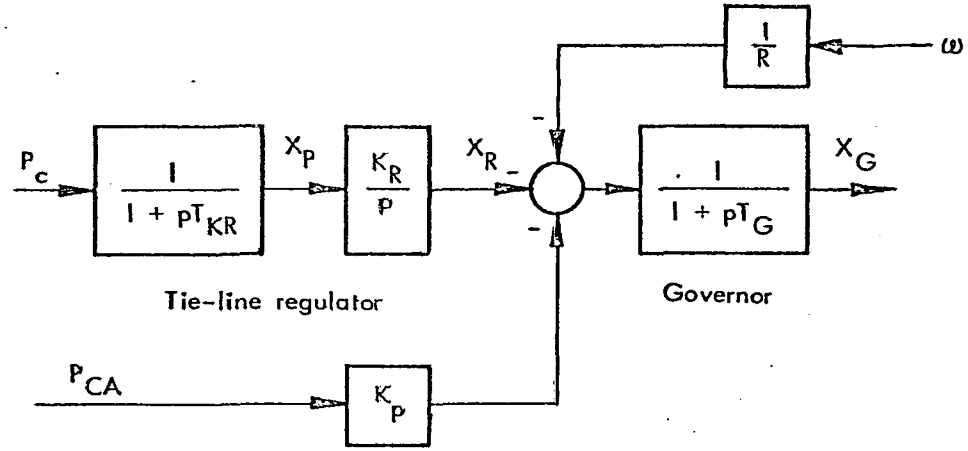
$$T_R \dot{X}_P + X_P = P_C$$

$$\dot{X}_P = \frac{P_C}{T_R} - \frac{X_P}{T_R} \tag{45}$$

$$T_G \dot{X}_G + X_G = -K_P P_{CA} - \frac{\omega}{R} - X_R \; ; \quad \omega = x_2$$

$$\dot{X}_G = -\frac{X_G}{T_G} - \frac{X_R}{T_G} - \frac{x_2}{RT_G} - \frac{K_P P_{CA}}{T_G} \tag{46}$$

These additions to the state-space formulation are given by Equation 47.



R = Steady-state speed regulation
K_P = Proportional gain of the regulator
K_R = Reset gain of the regulator
P_C = Reset control signal
P_{CA} = Proportional control signal
T_G = Governor time constant
T_{KR} = Time constant associated with reset control signal
X_G, X_P, X_R = Intermediate variables in block diagram

Figure 13. Block diagram of addition of tie-line controller

$$\begin{bmatrix} \dot{\delta} \\ \dot{X}_R \\ \dot{X}_P \\ \dot{X}_G \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & & K_R & \\ & & \frac{-1}{T_{KR}} & \\ \frac{-1}{RT_G} & \frac{-1}{T_G} & & \frac{-1}{T_G} \end{bmatrix} \cdot \begin{bmatrix} \omega \\ X_R \\ X_P \\ X_G \end{bmatrix} + \begin{bmatrix} & \\ & \\ & \frac{1}{T_{KR}} \\ \frac{-K_P}{T_G} & \end{bmatrix} \cdot \begin{bmatrix} P_{CA} \\ P_C \end{bmatrix} \qquad (47)$$

VIII. MULTI-MACHINE PROBLEM

The analysis can be extended to cover two or more machines with the use of matrix algebra. Undrill (30) uses the terminal voltage of one machine as reference. Laughton (21) uses a separate infinite busbar reference axis. See Figure 14. The extension to the two machines is included for completeness following Laughton.

A. n-Machine Analysis

The equations which relate the machine variables and acceleration are

$$v_{qi} = V_{ti} \cos \delta_{Ii}$$

$$v_{di} = V_{ti} \sin \delta_{Ii} \tag{48}$$

where $i = 1,2,\ldots n$ (for 2 machines, $n = 2$).

Now the armature current of a generator is a function of all the other armature currents, it is related to its direct and quadrature reference frame and the system reference axis herein called V_b for simplicity.

Currents are defined $I_i = a_i + jb_i$ with

$$I_i^2 = a_i^2 + b_i^2 \tag{49}$$

where a_i and b_i are variables satisfying network laws.

Now matrix equations for node-voltage network approach

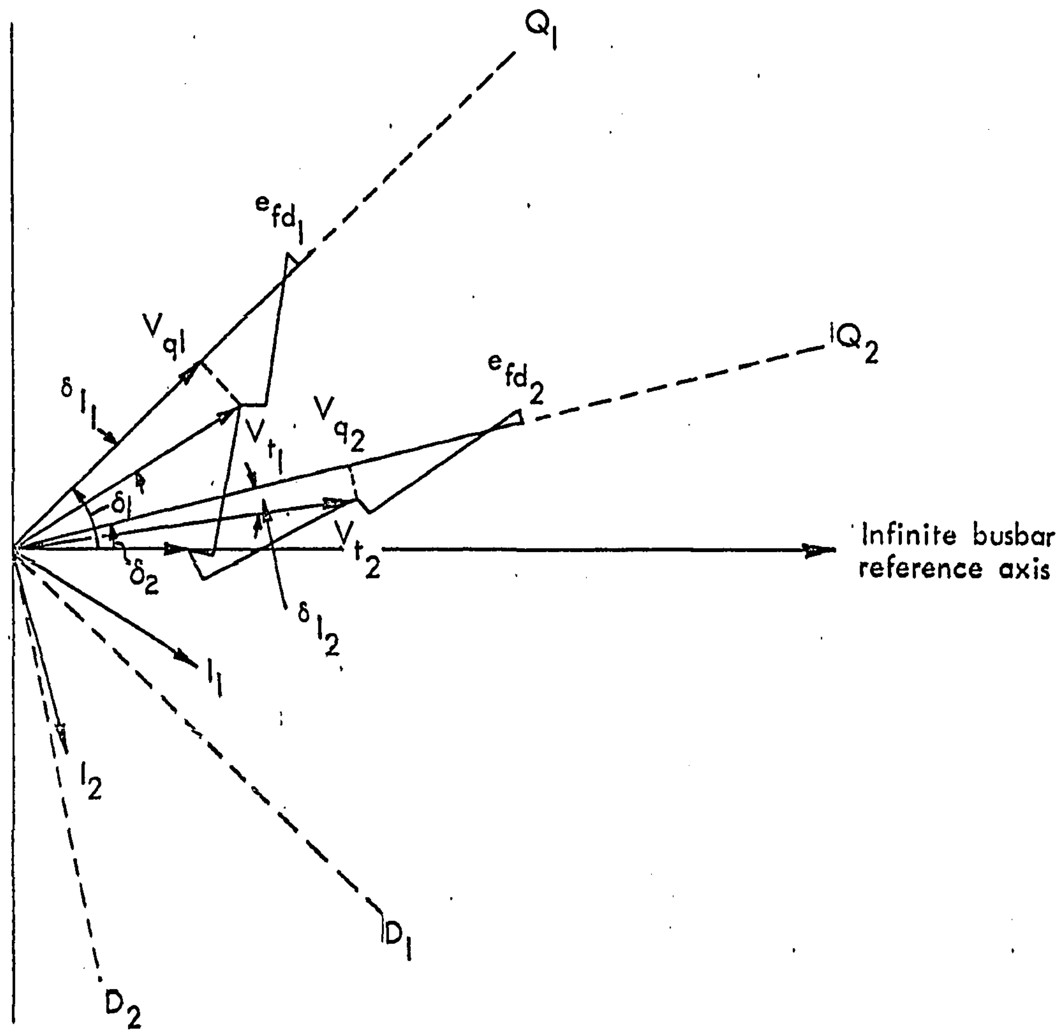


Figure 14. Phasor diagram of a two-machine system

are $\underline{I} = \underline{Y}\ \underline{V}$ where $\underline{Y}$ = network node admittance matrix and $\underline{V}$ the matrix of terminal voltages. Defining $\underline{Y} = \underline{G} - j\underline{B}$ and $\underline{v} = \underline{e} + j\underline{f}$;

$$a_i + jb_i = \sum_{m=1}^{n} (G_{im} - jB_{im})(e_m + jf_m),$$

where

$$e_m = V_{tm} \cos\ (\delta_m - \delta_{Im})$$

and

$$f_m = V_{tm} \sin(\delta_m - \delta_{Im})$$

therefore the final system equations are

$$a_i = \sum_{m=1}^{n} \{G_{im}V_{tm} \cos(\delta_m - \delta_{Im}) + B_{im}V_{tm} \sin(\delta_m - \delta_{Im})\}$$

$$b_i = \sum_{m=1}^{n} \{G_{im}V_{tm} \sin(\delta_m - \delta_{Im}) - B_{im}V_{tm} \cos(\delta_m - \delta_{Im})\} \tag{50}$$

The operating point may be selected by means of an iterative matrix analysis (29) and perturbation relationships developed about this point as before.

For two machines on an infinite bus, the perturbation equations in matrix form are shown in Equation 51.

The constants k_{ij} in Equation 51 are the constants involved in the perturbation equations corresponding to Equation 50; thus

$$
\left[\begin{array}{c}
\Delta t_{m1}\\
\Delta t_{m2}\\
\\
\\
\\
\\
\\
\hline
\\
\\
\\
\\
\\
\\
\\
\\
\\
\hline
\\
\\
\\
\\
\\
\\
\end{array}\right]
=
\left[\begin{array}{ccccccccccccccc}
M_1p^2 + D_1p & & & & & & & & & & -i_{q_1} & v_{d_1} & i_{q_1} & -i_{d_1} & \\
 & M_2p^2 + D_2p & & & & & & & & & & & & & \\
 & & & & & & & & & (1 + \tau_{f1}p) & -\tau_{f1}p(x_{d1} - x'_{d1}) & & & & \\
 & & & 1 & & & & -\frac{v_{d1}}{\nabla t_1} & -\frac{v_{q1}}{\nabla t_1} & & & & & & \\
 & & & & 1 & & & & & & & & & & \\
 & & & & & 1 & & & & & -\frac{i_{d1}}{I_1} & & & & \\
 & & & & & & 1 & & & & & -\frac{i_{q1}}{I_1} & & & \\
\hline
 & & & -\operatorname{Sin}\delta_{1_1} & & & & 1 & & & & & & & \\
 & & & -\operatorname{Cos}\delta_{1_1} & & & & & 1 & & & & & & \\
 & & & & & & & & 1 & -1 & x_{d1} & r_{q1} & & & \\
 & & 1 & & & & & & & -1 & x_{d1} - x'_{d1} & & & & \\
 & & & & & & & 1 & & & r_{d1} & -x_{q1} & & & \\
 & & & & & & & & & -1 & x_{d1} & & 1 & & \\
 & & & & & & & & & & & x_{q1} & & 1 & -\frac{a_1}{I_1} \\
-K_{22} & -K_{24} & & -K_{21} & -K_{23} & & & & & & & & & & 1 \\
-K_{12} & -K_{14} & & -K_{11} & -K_{13} & & & & & & & & & & \\
\hline
 & & & & -\operatorname{Sin}\delta_{1_2} & & & & & & & & & & \\
 & & & & -\operatorname{Cos}\delta_{1_2} & & & & & & & & & & \\
 & & 1 & & & & & & & & & & & & \\
 & & & & & & 1 & & & & & & & & \\
-K_{42} & -K_{44} & & -K_{41} & -K_{43} & & & & & & & & & & \\
-K_{32} & -K_{34} & & -K_{31} & -K_{33} & & & & & & & & & & \\
\end{array}\right]
$$

$$
\left[\begin{array}{cccccccccccc}
 & & & & & & & & & & & \\
 & & & & & -i_{q_2} & i_{d_2} & i_{q_2} & -i_{d_2}' & & & \\
 & & & & & & & & & & & \\
 & & & & (1+\tau_{f2}p) & -\tau_{f2}p(x_{d2}-x_{d2}') & & & & & & \\
 & & & & & & & & & & & \\
 & & -\frac{v_{d_2}}{V_{t_2}} & -\frac{v_{q_2}}{V_{t_2}} & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
\hline
 & -V_{t1}\cos\delta_{t_1} & & & & & & & & & & \\
 & V_{t1}\sin\delta_{t_1} & & & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
-\frac{b_1}{I_1} & & & & & & & & & & & \\
1 & & & & & & & & & & & \\
 & K_{22} & & & & & & & & & & K_{24} \\
 & K_{12} & & & & & & & & & & K_{14} \\
\hline
 & & 1 & & & & & & & & & -V_{t2}\cos\delta_{t_2} \\
 & & & 1 & & & & & & & & V_{t2}\sin\delta_{t_2} \\
 & & & 1 & -1 & x_{d2} & r_{q2} & & & & & \\
 & & & & -1 & x_{d2}-x_{d2}' & & & & & & \\
 & & 1 & & & r_{d2} & -x_{q2} & & & & & \\
 & & & & -1 & x_{d2} & & 1 & & & & \\
 & & & & & & x_{q2} & & 1 & & & \\
 & & & & & & & & & -\frac{a_2}{I_2} & -\frac{b_2}{I_2} & \\
 & K_{42} & & & & & & & & & 1 & K_{44} \\
 & K_{32} & & & & & & & & 1 & & K_{34}
\end{array}\right]
\left[\begin{array}{c}
\Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\psi_{fd1} \\ \Delta\psi_{fd2} \\ \Delta V_{t1} \\ \Delta V_{t2} \\ \Delta I_1 \\ \Delta I_2 \\
\hline
\Delta v_{d1} \\ \Delta v_{q1} \\ \Delta i_{fd1} \\ \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta\psi_{d1} \\ \Delta\psi_{q1} \\ \Delta a_1 \\ \Delta b_1 \\ \Delta\delta_{t_1} \\
\hline
\Delta v_{d2} \\ \Delta v_{q2} \\ \Delta i_{fd2} \\ \Delta i_{d2} \\ \Delta i_{q2} \\ \Delta\psi_{d2} \\ \Delta\psi_{q2} \\ \Delta a_2 \\ \Delta b_2 \\ \Delta\delta_{t_2}
\end{array}\right] \quad (51)
$$

$$\Delta a_j = \sum_k [\{G_{jk} \cos(\delta_k - \delta_{Ik}) + B_{jk} \sin(\delta_k - \delta_{Ik})\} \Delta V_{tk}$$

$$+ V_{tk}\{-G_{jk} \sin(\delta_k - \delta_{Ik}) + B_{jk} \cos(\delta_k - \delta_{Ik})\}(\Delta\delta_k - \Delta\delta_{Ik})]$$

$$\Delta b_j = \sum_k [\{G_{jk} \sin(\delta_k - \delta_{Ik}) - B_{jk} \cos(\delta_k - \delta_{Ik})\} \Delta V_{tk}$$

$$+ V_{tk}\{G_{jk} \cos(\delta_k - \delta_{Ik}) + B_{jk} \sin(\delta_k - \delta_{Ik})\}(\Delta\delta_k - \Delta\delta_{Ik})]$$

$$j,k = 1,2$$

or in matrix form

$$\begin{bmatrix} \Delta a_1 \\ \Delta b_1 \\ \Delta a_2 \\ \Delta b_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \cdot \begin{bmatrix} \Delta V_{t1} \\ (\Delta\delta_1 - \Delta\delta_{I1}) \\ \Delta V_{t2} \\ (\Delta\delta_2 - \Delta\delta_{I2}) \end{bmatrix} \qquad (52)$$

The coefficient matrix can be reduced by means of the Kron* reduction the same as for a single machine connected to an infinite bus. Writing the coefficient matrix in Equation 51 as

$$\underline{M} = \begin{bmatrix} M_1(p) + M_2 & M_3(p) + M_4 \\ M_5 & M_6 \end{bmatrix} \qquad (53)$$

where $M_1(p)$ and $M_3(p)$ contain terms involving the differential

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* Kron (20, p. 378).

operator p only. The reduced coefficient matrix representing the system of interest, represented by the first eight variables is

$$M_{\text{reduced}} = \{M_1(p) - M_3(p)M_6^{-1}M_5\} + \{M_2 - M_4M_6^{-1}M_5\} \quad (54)$$

The term in the last bracket and $M_3(p)M_6^{-1}M_5$ being found numerically on the digital computer. An equation similar to Equation 5 is then established and expanded to first-order differential-equation set. The rest of the analysis for the incorporation of control-equipment equations, dynamic performance, and stability limits follows as before.

This area of investigation leads to best ways for coordinating and interconnecting, control systems for dynamic stability for multi-machine operation. It is to this area which the experiences, confidences and simplifications gained with a single machine can fruitfully be extended and which are of ultimate interest and concern.

B. Two-Machine System Equations

The phasor diagram of a two-machine machine presentation is given in Figure 14. A separate reference axis is given. The generating units are described by Equation 51.

IX. REMARKS AND CONCLUSIONS

A. Machine Variables and A-constants

Machine variables and A-constants for the two values of tie-line reactances used in this study are given in Figures 15, 16, 17, and 18. Being in per-unit, as experience develops, these values will become increasingly helpful in predicting stable operation. Programs given in Appendix C.

B. Operation with Voltage Regulator

Figures 19, 20, 21, and 22 give dynamic stability with the voltage regulator only and at the nominal setting. Power output, machine damping and tie-line reactances are varied. Figure 27 summarizes these results.

Stability improves as unity power factor is approached from 0.90 power factor lagging with lighter power output and the lower tie-line reactance. This was as expected. The 0.95 lagging power factor full-load point was reported unstable at the higher tie-line reactance though close to being stable. Varying the damping had little effect on dynamic stability agreeing with Concordia's finding (6).

C. Operation with Voltage Regulator and Governor

Figures 23 , 24, 25, and 26 give dynamic stability under the same conditions as section B above except with both the governor and voltage regulator in operation. Figure 28 summarizes these results. An increase in range of stable operation to 0.975 power factor lagging was indicated.

It became evident that smaller increments of power factor should be taken at full-load output and the higher tie-line reactance. This evidenced stable operation up to and including 0.985 power factor lag, as given in Figure 30.

Lastly, it was desirable to ascertain if variations in voltage regulator settings would increase the power factor range on and above that given in Figure 29. Combinations of a few selected voltage regulator settings around the nominal setting produced results given in Figure 30. No further improvement over the nominal voltage regulator setting with the governor was evidenced. Computer time for this search was under 1.30 minutes.

D. General Observations

A derivative power signal was used for regulator damping. $dP/d\delta$ was determined from differentiation but it could have been derived through the use of A_7 and A_8 constants also.

The representation of loads under transients around an operating point needs investigation. Constant current or constant mva rather than constant impedance may be more realistic.

In a multi-machine system involving more equations, Kron's method of matrix reduction becomes a significant practical means for reducing the number of differential equations. The present eigenvalue routine handles a 30 x 30 matrix.

The margin between this dynamical approach to stability and a real-time steady-state study versus actual field tests is an area needing study and correlation.

By a combination of successive analysis, some synthesis, and judgment based on familiarity with both the problem and analytical approach, the system design and control schemes can be adjusted to give the maximum possible stability limit. VanNess (32) developed a program giving the sensitivity of eigenvalues with changes in system parameters to determine optimal design parameters and voltage regulator settings.

The new insights gained by the author in this study of the design and operation of modern-sized generating units has been marked. It has been stimulating to attempt to align academic thinking with present practices.

The method for extending the study to a two-machine system and ultimately n-machines using the multi-machine matrix approach, with iterated load flows, is included. This work has been left for future investigation. It is hoped valid simplifications in modeling will arise in this approach to stability. These are needed to counter the complexities associated with an expanding A-matrix resulting from increased state-space equations needed to express and interrelate a multi-machine system.

The following data reported as unstable (X) in Figure 27

0.950 pf lag, X-line = 0.718 pu, power = 1.000 pu, D = 2

0.950 pf lag, X-line = 0.718 pu, power = 1.000 pu, D = 3

0.975 pf lag, X-line = 0.414 pu, power = 1.000 pu, D = 2

are very close to stable and undoubtedly incorrect due to round-off error in finding the eigenvalues.

Figure 15. Machine variables for various armature power factors. Angles are in degrees and all other variables are in per-unit on machine rating base. Constant power output and constant 1.0 per-unit infinite bus voltage. Line reactance (X-line) = 0.414 pu.

Power factor	θ deg	δ_b deg	V_t	V_b	δ deg	I	I_d	I_q	V_d	V_q	ψ_d	ψ_q
1.000 pu power output												
0.900 lag	- 25.84	- 17.96	1.16	1.00	56.08	0.862	0.775	0.378	0.716	0.913	0.913	- 0.717
0.925	- 22.23	- 18.32	1.14	1.01	58.75	0.853	0.759	0.391	0.739	0.868	0.868	- 0.741
0.950	- 18.19	- 18.99	1.11	1.00	62.54	0.853	0.752	0.404	0.765	0.805	0.805	- 0.766
0.975	- 12.83	- 19.94	1.07	1.00	67.96	0.863	0.753	0.420	0.795	0.716	0.716	- 0.800
1.000	0	- 22.15	0.98	1.00	82.73	0.918	0.800	0.451	0.854	0.481	0.482	- 0.855
0.975 lead	12.83	- 26.24	0.85	1.00	105.13	1.086	0.992	0.441	0.834	0.164	0.065	- 0.836
0.950	18.19	- 27.74	0.80	1.02	115.03	1.184	1.106	0.423	0.799	0.038	0.039	- 0.801
0.833 pu power output												
0.900 lag	- 25.84	- 14.95	1.15	1.01	35.23	0.725	0.634	0.351	0.663	0.939	0.940	- 0.665
0.925	- 22.23	- 15.31	1.13	1.00	37.35	0.718	0.619	0.362	0.686	0.898	0.898	- 0.687
0.950	- 18.19	- 15.64	1.11	1.01	39.89	0.711	0.604	0.376	0.712	0.852	0.852	- 0.713
0.975	- 12.83	- 16.21	1.08	1.01	43.62	0.712	0.594	0.394	0.745	0.782	0.782	- 0.746
1.000	0	- 17.94	1.00	1.00	54.85	0.750	0.613	0.432	0.818	0.576	0.577	- 0.819
0.975 lead	12.83	- 20.10	0.91	1.00	70.44	0.845	0.714	0.453	0.858	0.305	0.306	- 0.859
0.950	18.19	- 21.49	0.86	1.00	79.11	0.918	0.802	0.351	0.845	0.162	0.163	- 0.846
0.667 pu power output												
0.900 lag	- 25.84	- 12.15	1.13	1.00	31.87	0.590	0.499	0.315	0.600	0.960	0.960	- 0.598
0.925	- 22.23	- 12.25	1.12	1.00	33.43	0.579	0.479	0.326	0.617	0.935	0.935	- 0.618
0.950	- 18.19	- 12.57	1.10	1.01	35.66	0.574	0.464	0.339	0.641	0.894	0.894	- 0.642
0.975	- 12.83	- 13.11	1.07	1.00	38.99	0.575	0.452	0.355	0.673	0.832	0.832	- 0.674
1.000	0	- 13.89	1.02	1.01	47.53	0.588	0.434	0.397	0.752	0.689	0.689	- 0.753
0.975 lead	12.83	- 15.24	0.95	1.00	60.48	0.648	0.479	0.436	0.827	0.468	0.469	- 0.828
0.950	18.19	- 15.83	0.92	1.00	67.40	0.687	0.520	0.448	0.849	0.353	0.354	- 0.850

Figure 16. Machine variables for various armature power factors. Angles are in degrees and all other variables are in per-unit on machine-rating base. Constant power output and constant 1.0 per-unit infinite bus voltage. Line reactance (X-line) = 0.718 pu.

Power factor	θ deg	δ_b deg	V_t	V_b	δ deg	I	I_d	I_q	V_d	V_q	ψ_d	ψ_q
1.000 pu power output												
0.900 lag	- 25.84	- 29.59	1.25	1.00	65.10	0.800	0.702	0.384	0.726	1.018	1.018	- 0.727
0.925	- 22.23	- 30.75	1.21	1.00	68.96	0.804	0.700	0.395	0.748	0.951	0.951	- 0.749
0.950	- 18.19	- 32.90	1.15	1.00	75.04	0.823	0.716	0.408	0.772	0.853	0.853	- 0.773
0.975	- 12.83	- 35.99	1.07	1.00	84.00	0.863	0.753	0.420	0.795	0.716	0.716	- 0.797
1.000	0	- 49.47	0.80	1.06	118.86	1.125	1.053	0.396	0.749	0.282	0.283	- 0.751
0.975 lead	12.83	- 43.11	0.80	1.19	124.68	1.154	1.075	0.418	0.791	0.117	0.118	- 0.793
0.950	18.19	- 40.74	0.80	1.25	128.04	1.184	1.106	0.423	0.799	0.038	0.038	- 0.801
0.833 pu power output												
0.900 lag	- 25.84	- 24.12	1.25	1.00	56.37	0.667	0.566	0.352	0.667	1.057	1.058	- 0.669
0.925	- 22.23	- 25.24	1.21	1.00	59.98	0.670	0.562	0.364	0.689	0.994	0.995	- 0.691
0.950	- 18.19	- 26.30	1.17	1.00	64.02	0.718	0.559	0.378	0.716	0.925	0.926	- 0.717
0.975	- 12.83	- 28.14	1.11	1.00	70.53	0.693	0.569	0.395	0.748	0.820	0.820	- 0.749
1.000	0	- 34.95	0.93	1.00	93.60	0.806	0.689	0.420	0.794	0.484	0.485	- 0.795
0.975 lead	12.83	- 38.64	0.80	1.09	116.07	0.962	0.868	0.413	0.781	0.174	0.175	- 0.782
0.950	18.19	- 36.69	0.80	1.14	119.72	0.987	0.893	0.420	0.794	0.097	0.098	- 0.796
0.667 pu power output												
0.900 lag	- 25.84	- 19.45	1.23	1.00	48.29	0.542	0.442	0.313	0.593	1.077	1.078	- 0.594
0.925	- 22.23	- 19.76	1.21	1.00	50.23	0.536	0.427	0.324	0.614	1.043	1.043	- 0.614
0.950	- 18.19	- 20.75	1.17	1.00	53.86	0.540	0.421	0.337	0.639	0.980	0.981	- 0.640
0.975	- 12.83	- 21.65	1.13	1.00	58.14	0.544	0.413	0.355	0.672	0.908	0.909	- 0.673
1.000	0	- 24.51	1.02	1.00	72.04	0.588	0.[illegible]	0.397	0.752	0.689	0.689	- 0.753
0.975 lead	12.83	- 30.97	0.84	1.00	99.51	0.733	0.605	0.413	0.781	0.307	0.308	- 0.783
0.950	18.19	- 31.77	0.80	1.04	108.56	0.789	0.674	0.411	0.779	0.183	0.183	- 0.780

Figure 17. Synchronous generator A-constants and operating angles δ in degrees versus various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. Line reactance (X-line) = 0.414 pu.

Power factor	δ deg	A1	A2	A3	A4	A5	A6	A7	A8
1.000 pu power output									
0.900 lag	56.08	- 1.263	- 1.210	- 1.750	- 3.121	- 0.006	0.436	1.104	1.190
0.925	58.75	- 1.252	- 1.246	- 1.810	- 3.121	- 0.012	0.422	1.125	1.176
0.950	62.54	- 1.228	- 1.287	- 1.877	- 3.121	- 0.025	0.402	1.145	1.165
0.975	67.96	- 1.193	- 1.335	- 1.957	- 3.121	- 0.047	0.371	1.165	1.154
1.000	82.73	- 1.081	- 1.428	- 2.109	- 3.121	- 0.127	0.272	1.194	1.151
0.975 lead	105.13	- 0.697	- 1.397	- 2.060	- 3.121	- 0.295	0.106	1.148	1.211
0.950	115.03	- 0.455	- 1.342	- 1.970	- 3.121	- 0.373	0.026	1.100	1.240
0.833 pu power output									
0.900 lag	50.18	- 1.186	- 1.120	- 1.622	- 3.121	- 0.026	0.453	1.037	1.157
0.925	52.66	- 1.175	- 1.154	- 1.680	- 3.121	- 0.023	0.440	1.055	1.140
0.950	55.53	- 1.169	- 1.196	- 1.748	- 3.121	- 0.018	0.425	1.074	1.120
0.975	59.83	- 1.158	- 1.249	- 1.835	- 3.121	- 0.007	0.401	1.094	1.099
1.000	72.78	- 1.113	- 1.364	- 2.024	- 3.121	- 0.046	0.319	1.127	1.077
0.975 lead	90.54	- 0.972	- 1.428	- 2.128	- 3.121	- 0.157	0.185	1.140	1.114
0.950	100.63	- 0.815	- 1.408	- 2.095	- 3.121	- 0.232	0.104	1.125	1.155
0.667 pu power output									
0.900 lag	44.018	- 1.070	- 1.006	- 1.458	- 3.121	- 0.051	0.471	0.951	1.116
0.925	45.676	- 1.070	- 1.038	- 1.512	- 3.121	- 0.053	0.462	0.967	1.090
0.950	48.230	- 1.063	- 1.077	- 1.575	- 3.121	- 0.051	0.450	0.981	1.063
0.975	52.093	- 1.053	- 1.127	- 1.658	- 3.121	- 0.046	0.431	0.995	1.034
1.000	61.415	- 1.068	- 1.253	- 1.863	- 3.121	- 0.027	0.374	1.018	0.967
0.975 lead	75.719	- 1.055	- 1.372	- 2.056	- 3.121	- 0.037	0.273	1.042	0.969
0.950	83.230	- 1.026	- 1.409	- 2.115	- 3.121	- 0.081	0.213	1.057	0.994

Figure 18. Synchronous generator A-constants and operating angles δ in degrees versus various armature power factors. Constant power output and constant 1.0 per unit infinite bus voltage. Line reactance (X-line) = 0.718 pu.

Power factor	δ deg	A1	A2	A3	A4	A5	A6	A7	A8
1.000 pu power output									
0.900 lag	65.10	- 0.931	- 0.996	- 1.357	- 2.507	- 0.055	0.556	0.837	0.810
0.925	68.96	- 0.901	- 1.025	- 1.404	- 2.507	- 0.070	0.537	0.847	0.803
0.950	75.04	- 0.841	- 1.054	- 1.452	- 2.507	- 0.101	0.506	0.854	0.801
0.975	84.00	- 0.747	- 1.084	- 1.501	- 2.507	- 0.155	0.437	0.856	0.806
1.000	118.86	- 0.145	- 1.026	- 1.410	- 2.507	- 0.451	0.240	0.770	0.872
0.975 lead	124.68	- 0.097	- 1.079	- 1.498	- 2.507	- 0.533	0.100	0.793	0.868
0.950	128.04	- 0.037	- 1.089	- 1.515	- 2.507	- 0.576	0.032	0.794	0.870
0.833 pu power output									
0.900 lag	56.37	- 0.920	- 0.915	- 1.247	- 2.507	- 0.007	0.578	0.788	0.780
0.925	59.98	- 0.892	- 0.943	- 1.293	- 2.507	- 0.017	0.561	0.800	0.771
0.950	6.402	- 0.869	- 0.976	- 1.347	- 2.507	- 0.029	0.540	0.807	0.759
0.975	70.53	- 0.827	- 1.017	- 1.414	- 2.507	- 0.057	0.504	0.816	0.752
1.000	93.60	- 0.614	- 1.076	- 1.510	- 2.507	- 0.204	0.355	0.811	0.786
0.975 lead	116.07	- 0.304	- 1.059	- 1.486	- 2.507	- 0.406	0.148	0.776	0.837
0.950	119.72	- 0.271	- 1.076	- 1.513	- 2.507	- 0.447	0.083	0.780	0.839
0.667 pu power output									
0.900 lag	48.29	- 0.856	- 0.814	- 1.107	- 2.507	- 0.028	0.598	0.725	0.747
0.925	50.23	- 0.850	- 0.840	- 1.149	- 2.507	- 0.028	0.589	0.735	0.727
0.950	53.86	- 0.828	- 0.871	- 1.201	- 2.507	- 0.022	0.572	0.742	0.711
0.975	58.14	- 0.813	- 0.913	- 1.269	- 2.507	- 0.013	0.549	0.749	0.688
1.000	72.04	- 0.767	- 1.015	- 1.433	- 2.507	- 0.038	0.461	0.759	0.667
0.975 lead	99.51	- 0.543	- 1.053	- 1.496	- 2.507	- 0.227	0.250	0.754	0.757
0.950	108.56	- 0.447	- 1.049	- 1.490	- 2.507	- 0.305	0.156	0.746	0.785

Figure 19. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.414 pu. Machine damping D = 2.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 27.063	- 196.716	- 26.894	- 196.695	- 26.856	- 196.691	- 26.826	- 196.688	- 26.789	- 196.683	- 26.783	- 196.675	- 26.780	- 196.713
- 27.063	196.716	- 26.894	196.695	- 26.856	196.691	- 26.826	196.688	- 26.789	196.683	- 26.783	196.675	- 26.780	196.713
- 145.595	0	- 145.674	0	- 145.686	0	- 145.704	0	- 145.760	0	- 145.954	0	- 145.980	0
- 1.094	- 0.407	- 0.872	0	- 0.932	0	- 1.074	- 0.165	- 0.871	0	- 0.868	0	- 0.965	0
- 1.094	0.407	- 1.242	0	- 1.201	0	- 1.074	0.165	- 1.288	0	- 1.195	0	- 1.096	0
- 0.006	0	- 0.005	0	- 0.015	0	- 0.017	0	- 0.0176	- 0.039	- 0.019	0	- 0.016	0
- 0.164	0	- 0.159	0	- 0.120	0	- 0.085	0	- 0.0176	0.039	0.081	0	0.101	0
Power output = 0.833 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)			
- 26.877	- 196.694	- 26.849	- 196.690	- 26.827	- 196.688	- 26.804	- 196.685	- 26.773	- 196.681	- 26.760	- 196.680	- 26.760	- 196.662
- 26.877	196.694	- 26.849	196.690	- 26.827	196.688	- 261804	196.685	- 26.773	196.681	- 26.760	196.680	- 26.760	196.662
- 145.708	0	- 145.714	0	- 145.716	0	- 145.722	0	- 145.745	0	- 145.815	0	- 145.894	0
- 0.805	0	- 0.766	0	- 0.935	0	- 0.909	0	- 1.072	- 0.051	- 0.941	0	- 0.893	0
- 1.241	0	- 1.286	0	- 1.148	0	- 1.193	0	- 1.072	0.051	- 1.197	0	- 1.202	0
- 0.004	0	- 0.002	0	- 0.010	0	- 0.014	0	- 0.033	- 0.026	- 0.001	- 0.035	- 0.012	0
- 1.195	0	- 0.187	0	- 0.146	0	- 0.117	0	- 0.033	0.026	- 0.001	0.035	0.047	0
Power output = 0.667 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 26.830	- 196.688	- 26.816	- 196.686	- 26.799	- 196.683	- 26.783	- 196.682	- 26.760	- 196.679	- 26.740	- 196.773	- 26.740	- 196.670
- 26.830	196.688	- 26.816	196.686	- 26.799	196.683	- 26.783	196.682	- 26.760	196.679	- 26.740	196.773	- 26.740	196.670
- 145.767	0	- 145.767	0	- 145.771	0	- 145.776	0	- 145.768	0	- 145.774	0	- 145.787	0
- 0.820	0	- 0.815	0	- 0.788	0	- 0.808	0	- 0.870	0	- 0.932	0	- 0.965	0
- 1.155	0	- 1.174	0	- 1.218	0	- 1.217	0	- 1.218	0	- 1.201	0	- 1.177	0
- 0.003	0	- 0.002	0	- 0.008	0	- 0.005	0	- 0.021	0	- 0.005	0	- 0.007	0
- 0.210	0	- 0.197	0	- 1.171	0	- 0.150	0	- 0.079	0	- 0.042	0	- 0.017	0

Figure 20. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values (K_A = 400, K_F = 0.06, K_D = 4, T_D = 1). Line reactance (X-line) = 0.414 pu. Machine damping D = 3.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)		(Stable)				(Stable)					
- 26.935	- 196.700	- 26.894	- 196.696	- 26.857	- 196.692	- 26.825	- 196.687	- 26.789	- 196.683	- 26.783	- 196.682	- 26.781	- 196.683
- 26.935	196.700	- 26.894	196.696	- 26.857	196.692	- 26.825	196.687	- 26.789	196.683	- 26.783	196.682	- 26.781	196.683
- 219.	0	- 219.038	0	- 219.046	0	- 219.058	0	- 219.096	0	- 219.224	0	- 219.241	0
- 0.847	0	- 0.790	0	- 0.779	0	- 0.784	0	- 0.775	0	- 0.794	0	- 0.847	0
- 1.083	0	- 1.147	0	- 1.171	0	- 1.176	0	- 1.209	0	- 1.137	0	- 1.085	0
- 0.003	0	- 0.003	0	- 0.008	0	0.001	0	- 0.014	- 0.036	- 0.013	0	- 0.014	0
- 0.139	0	- 0.128	0	- 0.102	0	- 0.089	0	- 0.014	0.036	0.061	0	- 0.080	0
Power output = 0.833 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 26.877	- 196.693	- 26.849	- 196.689	- 26.827	- 196.687	- 26.805	- 196.684	- 26.773	- 196.681	- 26.760	- 196.679	- 26.760	- 196.680
- 26.877	196.693	- 26.849	196.689	- 26.827	196.687	- 26.805	196.684	- 26.773	196.681	- 26.760	196.679	- 26.760	196.680
- 219.061	0	- 219.065	0	- 219.067	0	- 219.070	0	- 219.085	0	- 219.132	0	- 219.185	0
- 0.817	0	- 0.798	0	- 0.778	0	- 0.860	0	- 0.841	0	- 0.867	0	- 0.859	0
- 1.078	0	- 1.107	0	- 1.138	0	- 1.077	0	- 1.124	0	- 1.104	0	- 1.087	0
- 0.008	0	- 0.009	0	- 0.011	0	- 0.011	0	- 0.001	0	0.010	0	- 0.026	0
- 0.144	0	- 0.129	0	- 0.114	0	- 0.091	0	- 0.056	0	- 0.014	0	0.049	0
Power output = 0.667 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 26.830	- 196.688	- 26.816	- 196.687	- 26.800	- 196.684	- 26.783	- 196.682	- 26.760	- 196.680	- 26.745	- 196.678	- 26.741	- 196.677
- 26.830	196.688	- 26.816	196.687	- 26.800	196.684	- 26.783	196.682	- 26.760	196.680	- 26.745	196.678	- 26.741	196.677
- 219.100	0	- 219.099	0	- 219.102	0	- 219.105	0	- 219.100	0	- 219.103	0	- 219.114	0
- 0.930	- 0.119	- 0.932	- 0.091	- 0.938	- 0.093	- 0.946	- 0.113	- 0.796	0	- 0.819	0	- 1.013	0
- 0.930	0.119	- 0.932	0.091	- 0.938	0.093	- 0.946	0.113	- 1.131	0	- 1.144	0	- 0.009	- 0.011
- 0.009	0	- 0.005	0	- 0.007	0	- 0.012	0	- 0.016	0	- 0.005	0	- 0.009	0.011
- 0.140	0	- 0.137	0	- 0.122	0	- 0.098	0	- 0.063	0	- 0.034	0		

Figure 21. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.718 pu. Machine damping D = 2.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)				(Stable)							
- 27.063	- 196.716	- 26.954	- 196.703	- 26.880	- 196.693	- 26.826	- 196.687	- 26.817	- 196.685	- 26.809	- 196.685	- 26.807	- 196.677
- 27.063	196.716	- 26.954	196.703	- 26.880	196.693	- 26.826	196.687	- 26.817	196.685	- 26.809	196.685	- 26.807	- 196.677
- 145.84	0	- 145.852	0	- 145.881	0	- 145.929	0	- 146.414	0	- 146.457	0	- 146.500	0
- 0.952	0	- 0.950	- 0.387	- 0.947	0	- 0.760	0	- 0.680	0	- 0.680	0	- 0.662	0
- 0.952	0	- 0.950	- 0.387	- 0.947	0	- 1.102	0	- 0.993	0	- 0.996	0	- 1.005	0
- 0.007	0	- 0.004	0	0.002	- 0.440	- 0.017	- 0.031	0.007	0	0.011	0	- 0.003	0
- 0.080	0	- 0.073	0	- 0.055	0.440	- 0.017	0.031	0.253	0	0.297	0	0.344	0
Power output = 0.833 pu													
(Stable)		(Stable)		(Stable)		(Stable)							
- 26.979	- 196.705	- 26.902	- 196.700	- 26.85	- 196.69	- 26.811	- 196.685	- 26.772	- 196.681	- 26.781	- 196.681	- 26.781	- 196.682
- 26.979	196.705	- 26.902	196.700	- 26.85	196.69	- 26.811	196.685	- 26.772	196.681	- 26.781	196.681	- 26.781	196.682
- 145.841	0	- 145.856	0	- 145.87	0	- 145.890	0	- 145.996	0	- 146.316	0	- 146.347	0
- 0.632	0	- 0.750	0	- 0.747	0	- 0.729	0	- 0.782	0	- 0.677	0	- 0.701	0
- 1.180	0	- 1.088	0	- 1.10	0	- 1.123	0	- 1.059	0	- 1.026	0	- 0.996	0
- 0.001	0	- 0.016	0	- 0.019	0	- 0.009	0	0.005	- 0.032	0.001	0	0.007	0
- 0.172	0	- 0.117	0	- 0.091	0	- 0.078	0	0.005	0.032	0.192	0	0.207	0
Power output = 0.667 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 26.895	- 196.696	- 26.861	- 196.692	- 26.824	- 196.688	- 26.796	- 196.684	- 26.760	- 196.680	- 26.747	- 196.677	- 26.754	- 196.678
- 26.895	196.696	- 26.861	- 196.692	- 26.824	196.638	- 26.796	196.684	- 26.760	196.680	- 26.747	196.677	- 26.754	196.678
- 145.875	0	- 145.378	0	- 145.889	0	- 145.897	0	- 145.919	0	- 146.032	0	- 146.218	0
0.623	0	- 0.619	0	- 0.639	0	- 0.750	0	- 0.697	0	- 0.722	0	- 0.729	0
- 1.143	0	- [illegible]	0	- 1.153	0	- 1.067	0	- 1.147	0	- 1.100	0	- 0.993	0
- 0.008	0	- [illegible]	0	- 0.010	0	- 0.014	- 0.019	- 0.032	- 0.019	0.012	- 0.029	- 0.002	0
- 0.179	0	- 0.160		- 0.136	0	- 0.101	- 0.019	- 0.032	0.019	0.012	0.029	- 0.116	0

Figure 22. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.718 pu. Machine damping D = 3.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)				(Stable)							
- 27.063	- 196.716	- 26.954	- 196.703	- 26.875	- 196.694	- 26.825	- 196.687	- 26.817	- 196.686	- 26.808	- 196.686	- 26.810	- 196.706
- 27.063	196.716	- 26.954	196.703	- 26.875	196.694	- 26.825	196.687	- 26.817	196.686	- 26.808	196.686	- 26.810	196.706
- 219.146	0	- 219.156	0	- 219.176	0	- 219.208	0	- 219.530	0	- 219.559	0	- 219.588	0
- 0.637	0	- 0.696	0	- 0.633	0	- 0.680	0	- 0.631	0	- 0.638	0	- 0.642	0
- 1.094	0	- 1.045	0	- 1.109	0	- 1.067	0	- 1.000	0	- 1.000	0	- 0.989	0
- 0.002	0	- 0.004	0	0.001	0	- 0.013	0	0.001	0	0.004	0	- 0.001	0
- 0.101	0	- 0.079	0	- 0.063	0	- 0.013	0	0.181	0	0.213	0	0.240	0
Power output = 0.833 pu													
(Stable)		(Stable)		(Stable)		(Stable)							
- 26.980	- 196.707	- 26.903	- 196.697	- 26.852	- 196.691	- 26.812	- 196.686	- 26.772	- 196.680	- 26.782	- 196.682	- 26.780	- 196.684
- 26.980	196.707	- 26.903	196.697	- 26.852	196.691	- 26.812	196.686	- 26.772	196.680	- 26.782	196.682	- 26.780	196.684
- 219.150	0	- 219.159	0	- 219.167	0	- 219.181	0	- 219.252	0	- 219.466	0	- 219.484	0
- 0.617	0	- 0.622	0	- 0.697	0	- 0.632	0	- 0.767	0	- 0.658	0	- 0.653	0
- 1.089	0	- 1.095	0	- 1.035	0	- 1.101	0	- 0.969	0	- 0.989	0	- 1.000	0
- 0.004	0	- 0.028	0	- 0.041	0	- 0.033	- 0.023	0.004	- 0.043	- 0.019	0	0.033	0
- 0.119	0	- 0.076	0	- 0.041	0	- 0.033	0.023	0.004	0.043	0.153	0	0.125	0
Power output = 0.667 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 26.986	- 196.697	- 26.862	- 196.692	- 26.824	- 196.688	- 26.797	- 196.684	- 26.760	- 196.680	- 26.747	- 196.678	- 26.756	- 196.680
- 26.986	196.697	- 26.862	196.692	- 26.824	196.688	- 26.797	196.684	- 26.760	196.680	- 26.747	196.678	- 26.756	196.680
- 219.171	0	- 219.173	0	- 219.181	0	- 219.186	0	- 219.201	0	- 219.276	0	- 219.401	0
- 0.714	0	- 0.715	0	- 0.743	0	- 0.720	0	- 0.787	0	- 0.715	0	- 0.664	0
- 0.967	0	- 0.974	0	- 0.954	0	- 0.988	0	- 0.948	0	- 1.008	0	- 0.998	0
- 0.008	0	- 0.018	0	- 0.012	0	- 0.043	0	- 0.022	- 0.037	0.010	- 0.035	- 0.005	0
- 0.120	0	- 0.100	0	- 0.091	0	- 0.043	0	- 0.022	0.037	0.010	0.035	0.087	0

Figure 23. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.414 pu. Machine damping D = 2.

Real plus imaginary parts of eigenvalues vs power factors

Power output = 1.000 pu

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 0.100	0	- 0.100	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 6.667	0	- 6.667	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 10.000	0	- 10.000	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 26.934	- 196.706	- 26.329	- 196.698	- 26.856	- 196.693	- 26.825	- 196.691	- 26.790	- 196.687	- 26.783	- 196.687	- 26.788	- 196.688
- 26.934	196.706	- 26.329	196.698	- 26.856	196.693	- 26.825	196.691	- 26.790	196.687	- 26.783	196.687	- 26.788	196.688
- 145.669	0	- 145.676	0	- 145.686	0	- 145.703	0	- 145.760	0	- 145.953	0	- 146.074	0
- 0.999	0	- 1.000	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 1.110	0	- 1.123	0	- 1.134	0	- 1.146	0	- 1.158	0	- 1.066	0	- 1.002	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.010	0	- 0.001	0	- 0.002	0
- 0.170	0	- 0.152	0	- 0.129	0	- 0.100	0	- 0.026	0	0.066	0	0.125	0

Power output = 0.833 pu

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)			
- 0.100	0	- 0.100	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 6.667	0	- 6.667	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.1000	0	- 0.100	0
- 10.000	0	- 10.000	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 26.876	- 196.696	- 26.849	- 196.694	- 26.827	- 196.694	- 26.804	- 196.690	- 26.773	- 196.684	- 26.761	- 196.679	- 26.759	- 196.684
- 26.876	196.696	- 26.849	196.694	- 26.827	196.694	- 26.804	196.690	- 26.773	196.684	- 26.761	196.679	- 26.759	196.684
- 145.708	0	- 145.713	0	- 145.716	0	- 145.721	0	- 145.744	0	- 145.814	0	- 145.894	0
- 0.999	0	- 0.999	0	- 0.999	0	- 1.000	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 1.057	0	- 1.068	0	- 1.085	0	- 1.104	0	- 1.141	0	- 1.136	0	- 1.100	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.006	0	- 0.001	- 0.012	0.001	0
- 0.185	0	- 0.165	0	- 0.149	0	- 0.124	0	- 0.063	0	- 0.001	0.012	0.034	0

Power output = 0.667 pu

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 0.100	0	- 0.100	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 6.667	0	- 6.667	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 10.000	0	- 10.000	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 26.828	- 196.692	- 26.815	- 196.690	- 26.798	- 196.689	- 26.783	- 196.685	- 26.758	- 196.681	- 26.744	- 196.682	- 26.741	- 196.680
- 26.828	196.692	- 26.815	196.690	- 26.798	196.689	- 26.783	196.685	- 26.758	196.681	- 26.744	196.682	- 26.741	196.680
- 145.766	0	- 145.766	0	- 145.769	0	- 145.774	0	- 145.767	0	- 145.774	0	- 145.788	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 1.000	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.987	0	- 0.999	0	- 1.014	0	- 1.033	0	- 1.088	0	- 1.134	0	- 1.142	0
- 0.004	0	- 0.004	0	- 0.004	0	- 0.004	0	- 0.005	0	- 0.006	0	- 0.012	- 0.006
- 0.197	0	- 0.184	0	- 0.166	0	- 0.142	0	- 0.094	0	- 0.041	0	- 0.012	0.006

Figure 24. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.414 pu. Machine damping D = 3.

Real plus imaginary parts of eigenvalues vs power factors													
0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.933	- 196.700	- 26.892	- 196.697	- 26.845	- 196.692	- 26.824	- 196.689	- 26.788	- 196.685	- 26.783	- 196.682	- 26.787	- 196.684
- 26.933	196.700	- 26.892	196.697	- 26.845	196.692	- 26.824	196.689	- 26.788	196.685	- 26.783	196.682	- 26.787	196.684
- 219.034	0	- 219.039	0	- 219.047	0	- 219.057	0	- 219.095	0	- 219.223	0	- 219.034	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.933	0	- 0.943	0	- 0.954	0	- 0.965	0	- 0.982	0	- 0.932	0	- 0.893	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.013	0	- 0.001	0	- 0.002	0
- 0.134	0	- 0.120	0	- 0.102	0	- 0.078	0	- 0.016	0	0.050	0	0.093	0
Power output = 0.833 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.875	- 196.694	- 26.848	- 196.691	- 26.825	- 196.691	- 26.803	- 196.686	- 26.772	- 196.679	- 26.760	- 196.	- 26.758	- 196.682
- 26.875	196.694	- 26.843	196.691	- 26.825	196.691	- 26.803	196.686	- 26.772	196.679	- 26.760	196.	- 26.758	196.682
- 219.061	0	- 219.065	0	- 219.067	0	- 219.070	0	- 219.084	0	- 219.131	0	- 219.184	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.897	0	- 0.907	0	- 0.920	0	- 0.935	0	- 0.967	0	- 0.972	0	- 0.948	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.006	0	- 0.001	- 0.011	0.001	0
- 0.145	0	- 0.131	0	- 0.117	0	- 0.097	0	- 0.049	0	- 0.001	0.011	0.026	0
Power output = 0.667 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.827	- 196.690	- 26.814	- 196.688	- 26.798	- 196.686	- 26.781	- 196.683	- 26.758	- 196.678	- 26.743	- 196.677	- 26.740	- 196.677
- 26.827	196.690	- 26.814	196.688	- 26.798	196.686	- 26.781	196.683	- 26.758	196.678	- 26.743	196.677	- 26.740	196.677
- 219.100	0	- 219.101	0	- 219.102	0	- 219.105	0	- 219.[illegible]	0	- 219.104	0	- 219.113	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.[illegible]99	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.851	0	- 0.861	0	- 0.873	0	- 0.887	0	- 0.929	0	- 0.965	0	- 0.973	0
- 0.004	0	- 0.004	0	- 0.004	0	- 0.004	0	- 0.005	0	- 0.006	0	- 0.010	- 0.007
- 0.151	0	- 0.141	0	- 0.128	0	- 0.110	0	- 0.073	0	- 0.031	0	- 0.010	0.007

Figure 25. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values (K_A = 400, K_F = 0.06, K_D = 4, T_D = 1). Line reactance (X-line) = 0.718 pu. Machine damping D = 2.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)		(Stable)		(Stable)							
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 6.657	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 27.063	- 196.722	- 26.954	- 196.707	- 26.876	- 196.	- 26.326	- 196.690	- 26.801	- 196.686	- 26.790	- 196.687	- 26.787	- 196.688
- 27.063	196.722	- 26.954	196.707	- 26.876	196.	- 26.326	196.690	- 26.801	196.686	- 26.790	196.687	- 26.787	196.688
- 145.837	0	- 145.851	0	- 145.811	0	- 145.928	0	- 146.231	0	- 146.254	0	- 146.284	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.869	0	- 0.874	0	- 0.872	0	- 0.365	0	- 0.865	0	- 0.757	0	- 0.753	0
- 0.005	0	- 0.006	0	- 0.007	0	- 0.017	- 0.007	- 0.002	0	- 0.002	0	- 0.002	0
- 0.116	0	- 0.096	0	- 0.066	0	- 0.017	0.007	.149	0	0.186	0	0.212	0
Power output = 0.833 pu													
(Stable)		(Stable)		(Stable)		(Stable)							
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.978	- 196.711	- 26.901	- 196.703	- 26.851	- 196.694	- 26.810	- 196.691	- 26.771	- 196.684	- 26.768	- 196.686	- 26.765	- 196.684
- 26.978	196.711	- 26.901	196.703	- 26.851	196.694	- 26.810	196.691	- 26.771	196.684	- 26.768	196.686	- 26.765	196.684
- 145.842	0	- 145.856	0	- 145.867	0	- 145.889	0	- 145.995	0	- 146.151	0	- 145.	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.837	0	- 0.841	0	- 0.850	0	- 0.857	0	- 0.838	0	- 0.780	0	- 0.781	0
- 0.005	0	- 0.005	0	- 0.006	0	- 0.006	0	.004	- 0.012	- 0.001	0	- 0.001	0
- 0.143	0	- 0.124	0	- 0.104	0	- 0.074	0	.004	0.012	0.106	0	0.123	0
Power output = 0.667 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100		- 0.100	0	- 0.100	0
- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667		- 6.667	0	- 6.667	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000		- 10.000	0	- 10.000	0
- 26.894	- 196.699	- 26.681	- 196.695	- 26.823	- 196.691	- 26.796	- 196.689	- 26.759	- 196.683	- 26.746	- 196.682	- 26.745	- 196.681
- 26.894	196.699	- 26.681	196.695	- 26.823	196.691	- 26.796	196.689	- 26.759	196.683	- 26.746	196.682	- 26.745	196.681
- 145.875	0	- 145.877	0	- 145 889	0	- 145.	0	- 145.		- 145.	0	- 145.	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	.	- 0.999		- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999		- 19.999	0	- 19.999	0
- 0.783	0	- 0.792	0	- 0.801	0	- 0.815	0	- 0.847		- 0.819	0	- 0.800	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.006		0.004	0	0.001	0
- 0.164	0	- 0.152	0	- 0.133	0	- 0.111	0	- 0.054		0.019	0	0.054	0

Figure 26. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.718 pu. Machine damping D = 3.

Real plus imaginary parts of eignevalues vs power factors

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)		(Stable)		(Stable)							
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 6.667	0	- 6.667	0	- 5.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.657	0
- 27.061	- 196.717	- 26.952	- 196.705	- 26.874	- 196.693	- 26.825	- 196.686	- 26.800	- 196.684	- 26.790	- 196.682	- 26.787	- 196.683
- 27.061	196.717	- 26.952	196.705	- 26.874	196.693	- 26.825	196.686	- 26.800	196.684	- 26.790	196.682	- 26.787	196.683
- 219.147	0	- 219.157	0	- 219.	0	- 219.207	0	- 219.409	0	- 219.424	0	- 219.	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.738	0	- 0.743	0	- 0.746	0	- 0.745	0	- 0.678	0	- 0.689	0	- 0.658	0
- 0.006	0	- 0.005	0	- 0.008	0	- 0.013	- 0.009	- 0.002	0	- 0.002	0	- 0.002	0
- 0.091	0	- 0.074	0	- 0.050	0	- 0.013	0.009	0.109	0	0.136	0	0.155	0
Power output = 0.833 pu													
(Stable)		(Stable)		(Stable)		(Stable)							
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.976	- 196.709	- 26.900	- 196.700	- 26.850	- 196.691	- 26.810	- 196.686	- 26.771	- 196.680	- 26.767	- 196.683	- 26.764	- 196.681
- 26.976	196.709	- 26.900	196.700	- 26.850	196.691	- 26.810	196.686	- 26.771	196.680	26.767	196.683	- 26.764	196.681
- 219.151	0	- 219.160	0	- 219.167	0	- 219.182	0	- 219.252	0	- 219.356	0	- 219.367	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.713	0	- 0.719	0	- 0.727	0	- 0.735	0	- 0.732	0	- 0.701	0	- 0.703	0
- 0.005	0	- 0.006	0	- 0.006	0	- 0.006	0	0.002	- 0.011	- 0.001	0	- 0.001	0
- 0.112	0	- 0.096	0	- 0.080	0	- 0.057	0	0.002	- 0.001	0.078	0	0.091	0
Power output = 0.667 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.893	- 196.696	- 26.860	- 196.695	- 26.822	- 196.637	- 26.795	- 196.687	- 26.758	- 196.683	- 26.745	- 196.677	- 26.745	- 196.681
- 26.893	196.696	- 26.860	196.695	- 26.822	196.687	- 26.795	196.687	- 26.758	196.683	- 26.745	196.677	- 26.745	196.681
- 219.173	0	- 219.174	0	- 219.181	0	- 219.186	0	- 219.202	0	- 219.277	0	- 219.305	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 119.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.667	0	- 0.685	0	- 0.692	C	- 0.704	0	- 0.731	0	- 0.721	0	0.061	0
- 0.095	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.007	0	0.004	0	- 0.711	0
- 0.126	0	- 0.117	0	- 0.102	0	- 0.085	0	- 0.041	0	0.013	0	0.040	0

Line reactance	X-line = 0.414 pu						X-line = 0.718 pu					
damping	D = 2			D = 3			D = 2			D = 3		
power output, pu	1.000	0.833	0.667	1.000	0.833	0.667	1.000	0.833	0.667	1.000	0.833	0.667
power factor												
0.900 lag	S	S	S	S	S	S	S	S	S	S	S	S
0.925	S	S	S	S	S	S	S	S	S	S	S	S
0.950	S	S	S	S	S	S	X	S	S	X	S	S
0.975	S	S	S	X	S	S	S	S	S	S	S	S
1.000	S	S	S	S	S	S	X	X	S	X	X	S
0.975 lead	X	S	S	X	X	S	X	X	X	X	X	X
0.950	X	X	S	X	X	S	X	X	X	X	X	X

Figure 27. Summary of eigenvalue determination of dynamic stability (S = stable; X = unstable) for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$).

Line reactance	X-line = 0.414 pu						X-line = 0.718 pu					
damping	D = 2			D = 3			D = 2			D = 3		
power output, pu	1.000	0.833	0.667	1.000	0.833	0.667	1.000	0.833	0.667	1.000	0.833	0.667
power factor												
0.900 lag	S	S	S	S	S	S	S	S	S	S	S	S
0.925	S	S	S	S	S	S	S	S	S	S	S	S
0.950	S	S	S	S	S	S	S	S	S	S	S	S
0.975	S	S	S	S	S	S	S	S	S	S	S	S
1.000	S	S	S	S	S	S	X	X	S	X	X	S
0.975 lead	X	S	S	X	S	S	X	X	X	X	X	X
0.950	X	X	S	X	X	S	X	X	X	X	X	X

Figure 28. Summary of eigenvalue determination of dynamic stability (S = stable; X = unstable) for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values (K_A = 400, K_F = 0.06, K_D = 4, T_D = 1).

Machine damping	D = 2			D = 3		
power output, pu	1.000	0.833	0.667	1.000	0.833	0.667
power factor						
0.97 lag	S	S	S	S	S	S
0.98	S	S	S	S	S	S
0.99	X	S	S	X	S	S
1.00	X	X	S	X	X	S
0.99 lead	X	X	S	X	X	S
0.98	X	X	X	X	X	X

Figure 29. Summary of eigenvalue determination of dynamic stability (S = stable; X = unstable) for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.718 pu only and finer power factor increments from 0.970 lag.

Voltage regulator variables			
T_D	K_D	K_F	K_A
0.8	1	0.04	200
1.0	4	0.06	400
1.2	7	0.08	600

0.980 Lag	0.985	0.990	0.995	1.000
S	S	X	X	X

(All combinations of voltage regulator settings to the left resulted in stability determination as above.)

Figure 30. Summary of eigenvalue determination of dynamic stability (S = stable; X = unstable) for various armature power factors. Constant power output of 1.0 per-unit infinite bus voltage. With governor. Automatic voltage regulator set at various settings near nominal to scan for optimal regulator setting. Machine damping D = 2. Line reactance (X-line) = 0.718 pu.

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XII. APPENDIX A, LIST OF PRINCIPAL SYMBOLS

(Second column is the Fortran program name)

Generator and Transmission Line

Values are in pu on the machine base unless otherwise noted. Direct axis is denoted by d. Quadrature axis is denoted by q.

p.f.	PF	Power factor
r.f.	RF	Reactive factor
	SF	Complex of (PF,RF)
V_b	VB	Magnitude of infinite bus voltage
	VBR	Real part of VB
	VBJ	Imaginary part of VB
	VBB	Complex of (VBR,VBJ)
V_t	VT	Magnitude of generator terminal voltage; also real part of VT (VT taken as reference axis)
Y_{sh}	YS	Half of adjusted shunt admittance of the transmission line
Z_L	ZL	complex adjusted impedance of transmission line
I_{sh}	AIS	Complex current through YS
I_b	AIB	Complex current through ZL
	EPA	Complex voltage behind Potier reactance
E_p	EP	Magnitude of EPA
x_p	XP	Potier reactance
	AMPP	Complex generator current with respect to VT

I	AMP	Armature current, (magnitude of AMPP)
r_e	RE	Magnitude of the real part of ZL
x_e	XE	Magnitude of the imaginary part of ZL
r_a	RARM	Resistance of armature
r_d	RD	Resistance of armature, d-axis
r_q	RQ	Resistance of armature, q-axis
x_d	XD	Reactance of generator, d-axis
x_q	XQ	Reactance of generator, q-axis
x'_d	XDD	Transient reactance of generator
R_d	RRD	RD + RE
R_q	RRQ	RQ + RE
X_d	XXD	XD + XE
X_q	XXQ	XQ + XE
P	AMW	Power output, mva x p.f.
D	D	System damping, including effect of damper circuits
H		Inertia constant, mws/mva
f		Frequency, 60 HZ
M	AM	$H/\pi f$
τ_f	TF	Open-circuit generator-field time constant, sec.
τ'_{dz}	TTDZ	Effective field time constant, sec.
ϕ	PHIR	Angle between V_t and I, rad.
δ_b	DELBR	Angle between V_t and V_b, rad.
δ	DELR	Angle between V_b and q-axis, rad.
δ_q	DELQR	Angle between V_t and q-axis, rad.
δ_t	DELTR	Angle between I and q-axis, rad.

δ_I		Angle between V_t and infinite-bus ref. axis
I_d	AMPD	d-axis component of I
I_q	AMPQ	q-axis component of I
V_d	VD	d-axis component of V_t
V_q	VQ	q-axis component of V_t
ψ_d	PSID	d-axis component of flux-linkages
ψ_q	PSIQ	q-axis component of flux-linkages
ψ_f		Field flux linkages; $\psi_{fd} = \psi_f x_{fd}/x_f$

$A_1 \ldots A_1 \ldots$		Machine coefficients related to the operating point
x_f		Reactance of field winding
x_{fd}		Mutual reactance between field and d-axis armature winding
i_f		Field current; $i_{fd} = i_f x_{fd}$
v_f	VF	Field voltage
v_{fd}	EFD	$V_f x_{fd}/r_f$
r_f		Resistance of field winding
T_m		Mechanical torque
T_e		Electrical torque
T_a		Accelerating torque

p	d/dt
λ	Eigenvalues
$\underline{A}$	Matrices

$\underline{M}$		Matrices
$\underline{G}$		Matrices
$\underline{F}$		Matrices

Excitation System

K_A	AKA	Regulator gain
K_E	AKE	Exciter constant related to self-excited field
K_F	AKF	Regulator stabilizing circuit gain
S_E	SE	Exciter saturation function
T_A	TA	Regulator amplifier time constant, sec.
T_E	TE	Exciter time constant, sec.
T_F	TVF	Regulator stabilizing circuit time constant, sec.
T_R		Regulator input filter time constant, sec.
K_D	AKD	Power signal damping gain
T_D	TDA	Power signal damping time constant, sec.
V_R		Regulator output voltage
V_E	VE	Intermediate voltage point

Governor and Prime Mover

T_1	GT1	Control system time constant, sec.
T_2	GT2	Water hammer correction, sec.
T_3	GT3	Servo value motor time constant, sec.
T_4	GT4	Steam system time constant, sec.
T_5	GT5	Reheat steam system time constant, sec.
K_1	GK1	Overall gain (pu power/rad./sec.)
K_2	GK2	Fraction of steam reheated

XIII. APPENDIX B, SYSTEM EQUATIONS AND A-CONSTANTS OF THE SYNCHRONOUS GENERATOR

A. General Non-linear System Equations

Using Park's notation (25,26) with direct and quadrature-axis windings, the equations relating machine fluxes and currents are

$$\begin{bmatrix} \psi_{fd} \\ \psi_d \\ \psi_q \end{bmatrix} = \begin{bmatrix} 1 & -(x_d - x_d') & 0 \\ 1 & -x_d & 0 \\ 0 & 0 & -x_q \end{bmatrix} \cdot \begin{bmatrix} i_{fd} \\ i_d \\ i_q \end{bmatrix} \qquad (54)$$

where

$$x_d' \simeq x_L = x_d - \frac{x_{fd}^2}{x_f}$$

$$i_{fd} \equiv x_{fd} i_f$$

$$\psi_{fd} = \frac{x_{fd}}{x_f} \psi_f$$

ψ_{fd} is the variable representing the per-unit field flux linkages dependent upon the field/armature mutual reactance and field leakage reactance ratio.

The following assumes that voltages induced in the armature do not vary from voltages induced at synchronous speed, and that the armature voltages induced by the rate change of flux linkages during rotor oscillation are negligible compared to those generated by the synchronous rotating flux.

$$\begin{bmatrix} v_{fd} \\ v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 1+\tau_f p & -\tau_f p(x_d - x_d') & 0 \\ 0 & -r_d & x_q \\ 1 & -x_d & -r_q \end{bmatrix} \cdot \begin{bmatrix} i_{fd} \\ i_d \\ i_q \end{bmatrix} \qquad (55)$$

where

$$v_{fd} = \frac{x_{fd}}{r_f} v_f = (1+\tau_f p)\, i_{fd}$$

When the machine is considered to be connected to an infinite bus, proceeding from the internal machine conditions to that of the equivalent network, the following equations can be deduced from Figure 1.

$$\begin{aligned} v_d &= V_b \sin\delta + r_e i_d - x_e i_q \\ v_q &= V_b \cos\delta + x_e i_d + r_e i_q \end{aligned} \qquad (56)$$

The restraints on the change of rotor position, due to machine inertia and system damping, are expressed as

$$Mp^2 = T_a = T_m - T_e$$

or, more generally,

$$(Mp^2 + Dp)\delta = T_m - (\psi_d i_q - \psi_q i_d) \qquad (57)$$

and the terminal conditions

$$
\begin{aligned}
V_t^2 &= v_d^2 + v_q^2 \\
I^2 &= i_d^2 + i_q^2
\end{aligned}
\qquad (58)
$$

The above equations define the state of the machine with no control loops present.

B. A-constants

The A-constants derived in Equation 4 and given in Equation 5 are as follows from Laughton (21):

$$A_1 = \left(\frac{V_b}{-R_dR_q - X_qX_d'}\right)\{(\psi_q + i_qx_d')(R_q \cos\delta - X_q \sin\delta)$$

$$+ (\psi_d + i_dx_q)(R_d \sin\delta + X_d' \cos\delta)\}$$

$$A_2 = \left(\frac{1}{-R_dR_q - X_qX_d'}\right)\{\psi_d + i_dx_q)R_d + i_q(R_dR_q + X_qx_e) - \psi_qX_q\}$$

$$A_3 = \left(\frac{1}{-R_dR_q - X_qX_d'}\right)\{-V_b(x_d - x_d')(R_q \cos\delta - X_q \sin\delta)\}$$

$$A_4 = \frac{R_dR_q + X_dX_q}{-R_dR_q - X_qX_d'}$$

$$A_5 = \frac{1}{-R_dR_q - X_qX_d'}\,\frac{V_b}{V_t}[-v_d\{(x_qX_d' + r_dR_q) \cos\delta + (r_ex_q - r_dx_e) \sin\delta\}$$

$$- v_q\{(r_ex_d' - x_er_q) \cos\delta + (-x_d'X_q - r_qR_d) \sin\delta\}]$$

$$A_6 = \left(\frac{1}{-R_dR_q - X_qX_d'}\right)\left\{\frac{-v_d}{V_t}(r_ex_q - r_dx_e) - \frac{v_q}{V_t}(r_eR_d + x_eX_q)\right\}$$

$$A_7 = \left(\frac{1}{-R_dR_q - X_qX_d'}\right)V_b\left\{\frac{-i_d}{I}(-R_q \cos \delta + X_q \sin \delta)\right.$$

$$\left.\frac{-i_q}{I}(X_d' \cos \delta + R_d \sin \delta)\right\}$$

$$A_8 = \frac{1}{-R_dR_q - X_qX_d'}\ \frac{-i_d}{I}\ X_q - \frac{i_q}{I}\ R_d$$

Also, the transient field time constant defined by

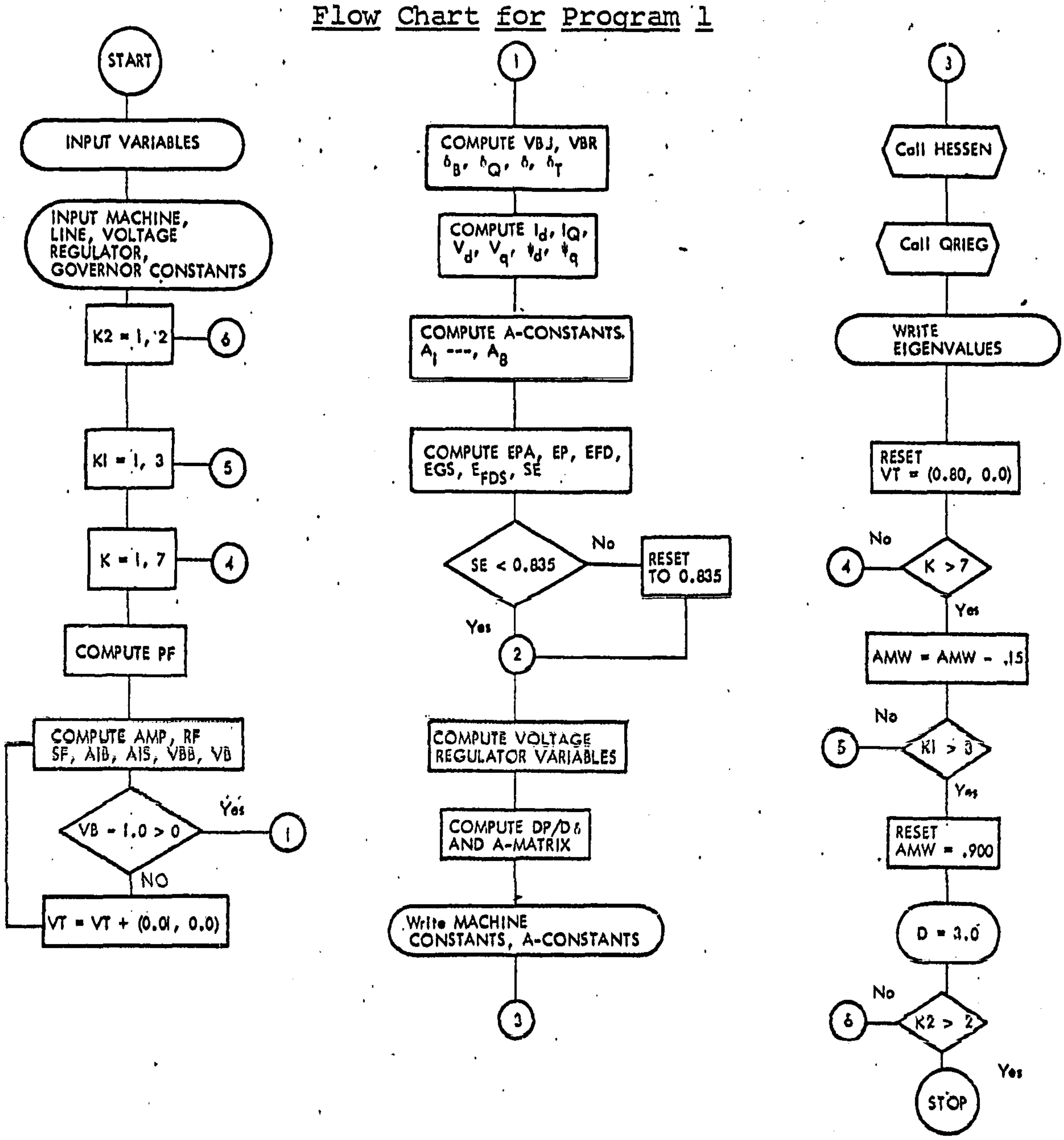
$$\tau_{dz}' = \left(\frac{R_dR_q + X_qX_d'}{R_dR_q + X_qX_d}\right)\tau_f$$

reduces the second row, second column diagonal element in Equation 4 to

$$-A_4(1 + \tau_{dz}'p). \tag{59}$$

XIV. APPENDIX C, PROGRAMS FOR A-MATRICES AND EIGENVALUES FROM COMPONENT DATA

Flow Chart for Program 1



```
C       PROGRAM 1
C
C       DETERMINING SYSTEM STABILITY BY EIGENVALUES.
C       WITH GOVERNOR, X-LINE=.414 PU; V-BUS=1.0 PU.
C       VOLTAGE REGULATOR SET AT NOMINAL VALUES.
C       POWER OUTPUT AT 1.0, .833 AND .667 PU.
C       MACHINE DAMPING AT D=2 AND D=3.
C       .90 PF LAG TO .95 LEAD IN .025 INCREMENTS.
C       FOR LAG PF, RF IS (-). FOR LEAD , RF IS (+).
C
        DIMENSION A(30,30), ROOTR(30), ROOTI(30)
        COMPLEX*8 VBB,VT,ZL,SF,YS,AIS,AIB,EPA,XP,AMPP
        READ(1,990) ZL,VT,AMW,YS
    990 FORMAT(7F10.4)
        XD=1.918
        XQ=1.896
        XP=(0.,.35)
        XDD=.333
        RQ = .0018
        RD = .0018
        RARM=.0018
C
C       C1 START.  INITIAL VALUES CORRESPONDING TO
C       X-LINE=.414 PU.
        VT=(1.15, 0.)
        ZL=(.027,.414)
        YS=(0.,.062)
        RE=.027
        XE=.414
        RRD=.029
        RRQ=.029
        XXD=2.332
        XXQ=2.310
        XXDD=.747
C       C1 END.
C
        AMW=.9000
        AM=.01367
        D=2.0
        TF=4.83
C       VOLTAGE REGULATOR CONSTANTS
        AKE=1.
        AKT=.10
        TA=.02
        TE=.51
        TVF=1.
C
C       C2 START.  VOLTAGE REGULATOR NOMINAL VALUES.
        AKA=400.
        AKF=.06
        AKD=4.
```

```
      TDA=1.
C     C2 END.
C
C     GOVERNOR
      GT1=.15
      GT2=0.
      GT3=.05
      GT4=.10
      GT5=10.
      GK1=.0443
      GK2=.23
C
C     C3 START
      DO 22 K2=1,2,1
      DO 21 K1=1,3,1
C     C3 END.
C
C     C4 START.
      DO 11 K=1,7,1
      IF (K-4) 2,2,3
    2 AI = K
      PF=.875+.025*AI
      PHIR = -ARCOS(PF)
      PHID = 180. * PHIR / 3.141593
      GO TO 4
    3 AI = K
      PF=1.125-.025*AI
C     C4 END.
C
      PHIR = ARCOS(PF)
      PHID = 180. * PHIR / 3.141593
    4 AMP=AMW/(PF*VT)
      RF = SIN (PHIR)
      SF = CMPLX(PF,RF)
      AIS=VT*YS
      AIB=AMP*SF-AIS
      VBB=VT-AIB*ZL
      VB =CABS(VBB)
      IF (VB-1.0) 13,10,10
   13 VT=VT+(.01,0.)
      GO TO 4
   10 VBR=REAL(VBB)
      VBJ=AIMAG(VBB)
      AMPP=AMP*SF
      DELBR = ATAN2(VBJ,VBR)
      DELBD = 180. * DELBR/3.141593
      Y=AMP*XQ*PF+AMP*RARM*RF
      Z=CABS(VT)+AMP*RARM*PF-AMP*XQ*RF
      DELQR=ATAN2(Y,Z)
      DELQD     = 180. * DELQR   /3.141593
      DELR =-DELBR + DELQR
```

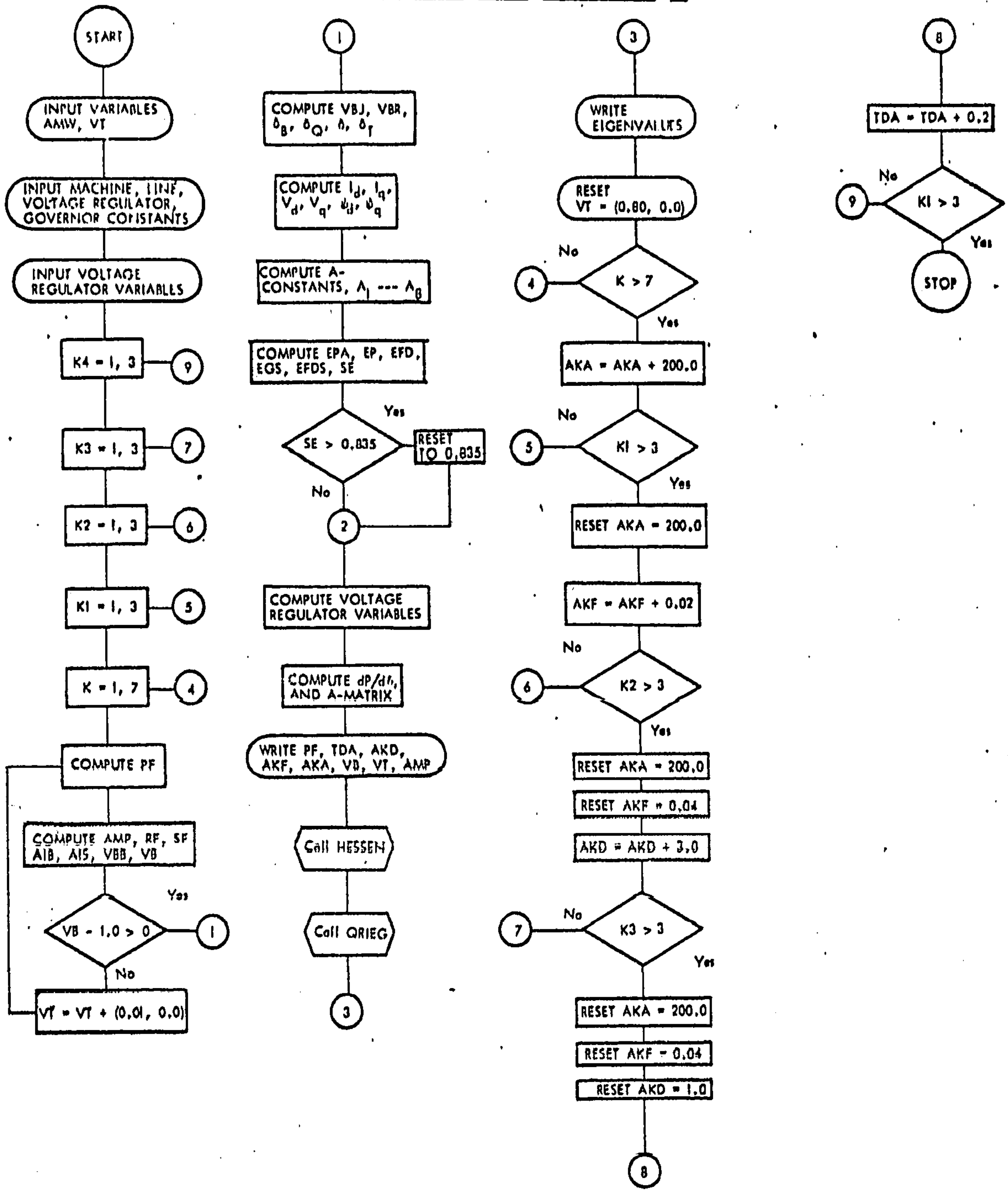
```
      DELD =-DELBD + DELQD
      DELTR     =-PHIR + DELQR
      DELTD     = 180. * DELTR    /3.141593
      AMPQ=AMP*COS(DELTR)
      AMPD=AMP*SIN(DELTR)
      VD=SIN(DELQR)*CABS(VT)
      VQ=COS(DELQR)*CABS(VT)
      PSIQ=-AMPQ*XQ
      PSID=VQ+AMPQ*RARM
      A1=(VB/(-RRD*RRQ-XXQ*XXDD))*((PSIQ+AMPQ*XDD)
     1(RRD*COS(DELR)-XXQ*SIN(DELR))+(PSID+AMPD*XQ)
     2*(RRD*SIN(DELR)+XXDD*COS(DELR)))
      A2= 1./(-RRD*RRQ-XXQ*XXDD)*((PSID+AMPD*XQ)*
     1RRD+AMPQ*(RRD*RRQ+XXQ*XE)-PSIQ*XXQ)
     1(RRQ*COS(DELR)-XXQ*SIN(DELR)))
      A3= 1./(-RRD*RRQ-XXQ*XXDD)*(-VB*(XD-XDD)*
      A4=(RRD*RRQ+XXD*XXQ)/(-RRD*RRQ-XXQ*XXDD)
      A5= 1./(-RRD*RRQ-XXQ*XXDD)* VB/VT *(-VD*((XQ
     1*XXDD+RD*RRQ)*COS(DELR)+(RE*RQ-RD*XE)*SIN
     2(DELR))-VQ*((RE*XDD-XE*RQ)*COS(DELR)+(-XDD*
     3XXQ-RQ*RRD)*SIN(DELR)))
      A6=1./(-RRD*RRQ-XXQ*XXDD)*(-VD/VT*(RE*RQ-RD*
     1XE)-VQ/VT*(RE*RRD+XE*XXQ))
     1(-RRQ*COS(DELR)+XXQ*SIN(DELR))-(AMPQ/AMP)*
      A7=1./(-RRD*RRQ-XXQ*XXDD)*VB*((-AMPD/AMP)*
     2(XXDD*COS(DELR)+RRD*SIN(DELR)))
      A8=1./(-RRD*RRQ-XXQ*XXDD)*(-AMPD/AMP)*XXQ-
     1(AMPQ/AMP)*RRD
      TTDZ=(RRD*RRQ+XXQ*XXDD)*(TF)/(RRD*RRQ+XXQ*
     1XXD)
      E=PSID+XDD*AMPD
      EPA=VT+AMP*SF*XP
      EP=CABS(EPA)
      EFD=VQ+AMPQ*RARM+AMPD*XD
      EGS=.035*EXP(7.358*(EP-.8))
      EFDS=EFD+EGS
      SE=.235*EXP(.214*(EFDS-.8))
      IF (SE-.835) 40,41,41
   40 GO TO 42
   41 SE=.835
   42 CONTINUE
C     VOLTAGE REGULATOR
      AV1=(TE*(TA+TVF)+TA*TVF*(AKE+SE))/(TA*TE*
     1TVF)
      AV2=((AKE+SE)*(TA+TVF)+TE+AKA+AKF)/(TA*TE*
     1TVF)
      AV3=(AKE+SE)/(TA*TE*TVF)
      BV2=AKA/(TA*TE)
      BV3=AKA/(TA*TE*TVF)
      D2LQR=2.*DELQR
      DPD=(E*VT*COS(DELQR))/XDD+((XDD-XQ)*VT*VT*
```

```
      1COS(D2LQR))/(XDD*XQ)
C       GOVERNOR
        GBP1=GK1*(GT1-GT2)/(GT1*(GT1-GT3))
        GBP2=GK1*(GT3-GT2)/(GT3*(GT3-GT1))
        M=11
        DO 17 I=1,11
        DO 17 J=1,11
     17 A(I,J)=0.
        A(1,2)=1.
        A(2,1)=A1/AM
        A(2,2)=-D/AM
        A(2,3)=A2/AM
        A(3,1)=-A3/(A4*TTDZ)
        A(3,3)=-1./TTDZ
C       VOLTAGE REGULATOR
        A(3,4)=-1./(A4*TTDZ*274.)
        A(4,4)=-AV1
        A(4,5)=1.
        A(5,4)=-AV2
        A(5,6)=1.
        A(5,7)=BV2
        A(6,4)=-AV3
        A(6,7)=BV3
        A(7,1)=-A5/TDA
        A(7,2)=(AKT*AKD*DPD)/TDA
        A(7,3)=-A6/TDA
        A(7,7)=-1./TDA
C       GOVERNOR
        A(8,8)=-1./GT4
        A(8,10)=-GBP1
        A(8,11)=-GBP2
        A(9,9)=-1./GT5
        A(9,10)=-GBP1
        A(9,11)=-GBP2
        A(10,2)=-1.
        A(10,10)=-1./GT1
        A(11,2)=-1.
        A(11,11)=-1./GT3
C
C       C5 START.
        WRITE(3,991)PF,RF,VT,SF,AMPP,AIS,AIB
    991 FORMAT(@0@,12F10.5)
        WRITE(3,991)VBB,VB,VBR,VBJ,DELBD,Y,Z,DELQD,
      1DELD,DELTD,AMP
        WRITE(3,991)AMPD,AMPQ,VD,VQ,PSID,PSIQ,YS,ZL
        WRITE(3,991)EPA,EP,EFD,EGS,EFDS,SE,D2LQR,DPD,D,AMW
        WRITE(3,991)A1,A2,A3,A4,A5,A6,A7,A8
        WRITE(3,992)AV1,AV2,AV3,BV2,BV3
    992 FORMAT(@0@,5F20.1)
C       C5 END.
```

```
      WRITE(3,993)PF,TDA,AKD,AKF,AKA,VB,VT,AMP,
     1AMW,D,XE
  993 FORMAT(@0@,12F10.5)
C
      CALL HESSEN (A,M)
      IPRNT=0
      CALL QREIG(A,M,ROOTR,ROOTI,IPRNT)
      WRITE(3,8) (ROOTR(I), ROOTI(I), I=1,M)
    8 FORMAT(@ @,2F10.5)
   11 VT=(.80,0.)
C
C     C6 START
      AMW=AMW-.1500
   21 CONTINUE
      AMW=.9000
      D=3.
   22 CONTINUE
C     C6 END.
      END
```

```
C      PROGRAM 2
C
C      SAME AS PROGRAM 1 EXCEPT X-LINE = .718  PU.
C
C      REMOVE CARDS C1 START THROUGH C1 END.
C      REPLACE BY CARDS AS FOLLOWS.
C
C      C1 START.
       VT=(1.25, 0. )
       ZL=(.046,.718)
       YS=(0.,0.106)
       RE=.046
       XE=.718
       RRD=.0502
       RRQ=.0502
       XXD=2.636
       XXQ=2.614
       XXDD=1.051
C      C1 END.
C
       END
```

Flow Chart for Program 3



```
C       PROGRAM 3
C       PROGRAM TO SCAN EIGENVALUES FOR STABILITY.
C       WITH GOVERNOR, D=2, X-LINE=.718 PU.
C       POWER OUTPUT=1.0 PU, V-BUS=1.0 PU.
C
C       VOLTAGE REGULATOR VARIED IN INCREMENTS AS
C       FOLLOWS.
C          TDA=.8, 1.0, 1.2
C          AKD=1.0, 4.0, 7.0
C          AKF=.04, .06, .08
C          AKA=200., 400., 600.
C
C       REMOVE CARDS C2 START THROUGH C2 END.
C       REPLACE BY CARDS AS FOLLOWS.
C
C       C2 START.
C       VOLTAGE REGULATOR VARIABLES INITIALIZATIONS
        TDA=.8
        AKD=1.0
        AKF=.04
        AKA=200.
C       C2 END
C
C       REMOVE CARDS C3 START THROUGH C3 END.
C       REPLACE BY CARDS AS FOLLOWS.
C
C       C3 START
        DO 24 K4=1,3,1
        DO 23 K3=1,3,1
        DO 22 K2=1,3,1
        DO 21 K1=1,3,1
C       C3 END.
C
C       REMOVE CARDS C4 START THROUGH C4 END.
C       (THIS DATA HAS BEEN FOUND IN PROGRAMS 1+2).
C       REMOVE CARDS C5 START THROUGH C5 END.
C       REPLACE BY CARDS AS FOLLOWS.
C
C       C5 START.
     21 AKA=AKA+200.
        AKA=200.
     22 AKF=AKF+.02
        AKA=200.
        AKF=.04
     23 AKD=AKD+3.
        AKA=200.
        AKF=.04
        AKD=1.0
     24 TDA=TDA+.2
C       C5 END.
        END
```

```
C       PROGRAM 4
C       PROGRAM TO OBTAIN .005 INCREMENT SCANNING OF
C       POWER FACTORS THROUGH .98 LAG PF TO 1.00 .
C
C       REMOVE CARDS C4 START THROUGH C4 END.
C       REPLACE BY CARDS AS FOLLOWS.
C
C       C4 START
        DO 11 K=1,5,1
        IF (K-4) 2,2,3
      2 AI=K
        PF=.975+.005*AI
        PHIR=-ARCOS(PF)
        PHID=180. * PHIR/ 3.141593
        GO TO 4
      3 AI=K
        PF=1.025-.005*AI
C       C4 END.
C
        END
```